



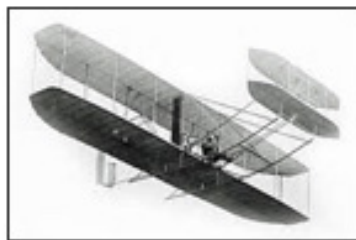
XXIII.

**PERSPECTIVE ON  
MATERIALS FAILURE  
THEORY AND APPLICATIONS**

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**1. Why It Matters**



**1903  
Wright Flyer**



**1967  
Saturn V**



**2030  
2040  
2050**

Would you be willing to cross a narrow footbridge over a deep ravine that was conceived and constructed by a friendly neighbor in his spare time? In a not greatly different situation, would you have been willing to go up with the Wright brothers on one of their very first experimental “aeroplane” flights? One such early passenger lost his life from a materials related failure and consequent crash with them. In addition to being an aeronautical genius of high order, Wilbur Wright was also heroic, McCullough, [1]. Would you or I be so heroic? For most people, prudence and caution would dictate carefully measured answers to such questions and situations, whether real or hypothetical.

Safety has usually been the prime consideration in any new physical project or endeavor. Reliability and dependability usually come in as a close second in priorities. This has transpired not just for hundreds of years but for thousands of years. However, it is only in the past few hundreds of years that the scientific method and approach has been brought to bear on the everlasting problem of materials failure.

Despite the many marvelous and far reaching achievements of physical science, a comprehensive understanding of materials failure has not been among them. This has not been for lack of interest or effort. Many of the most prominent and famous scientists in the past glory age of new and first discoveries took a run at the problem of materials failure. The second or third greatest physicist of all time, James Clerk Maxwell, did so and at least gained a foothold on the problem, but no general solution in the way that he so brilliantly succeeded with electricity and magnetism.

So the failure enigma has endured. In fact it has endured and been static for so long that there now is a strong skepticism as to the viability of a possible general theory of materials failure. In terms of current educational course offerings and from all the norms of usage, the outlook is not good. Overall, the preponderance of professional opinion on the prospects for improvement is deeply and overwhelmingly negative.

Is it hopeless? There is one source that offers at least a mild degree of optimism. This would be the somewhat related field of fracture mechanics. Fracture mechanics deals with the effect of a (usually) single dominant flaw in a structure required to bear load. In contrast, the field of materials failure applies to macroscopically homogeneous materials in any state of three dimensional stress using what are commonly called failure criteria. Fracture mechanics has been a resounding success. Could fracture mechanics somehow supply the template for failure criteria development? The answer always has been a firm and decisive no, failure criteria do not follow by mimicking the steps of fracture mechanics. Nevertheless fracture mechanics does provide the inspiration for traversing the mountain of historical misinformation and ill conceived approaches for failure criteria that has completely obscured the technical landscape.

As of now, the field continues to limp along using outdated and demonstrably incorrect failure criteria that came from the distant past. The

situation has nearly reached the point of total negligence. Will it ever be corrected? If not, what will be the consequences? If so, then how long before reform: years, decades, what? The place to begin is with a closer look at the unusual, even strange history of trying to develop failure criteria, in order to see what didn't work and why it didn't work.

## 2. Some Quick History



The search for failure criteria for homogeneous and isotropic materials goes back almost to the beginning of mechanics. The original conception of Coulomb [2] in the late 1700's was that the shear stress  $\tau$  on the failure surface is related to the normal stress  $\sigma$  acting across the failure surface as

$$\tau \leq c - \mu\sigma \quad (1)$$

where the two parameters  $c$  and  $\mu$  are material specific. With  $\mu=0$  this is just the maximum shear stress criterion commonly known today as the Tresca criterion, but it was really Coulomb who first recognized it.

Coulomb was a brilliant engineer working at military installations in early adulthood and much concerned with structures and stability, possibly of soil embankments. In fact he labeled the parameter  $\mu$  as the coefficient of internal friction, suggestive of granular materials flow or incipient flow. This is the simplest form that relates shear stress on the failure plane to transverse normal stress, tensile or compressive. Thus the failure criterion is assumed to depend upon the tractions acting across the failure surface and to be independent of the stress components in the plane of the assumed failure surface.

The failure criterion was semi- successful over the next many years but not successful enough to believe it had generality. So the search continued. There was considerable optimism that the existing very high levels of physical insight combined with some ingeniously clever testing

would uncover the treasure of a simple but universal failure criterion. Many of history's greatest scientists took part in the search. It was not successful.

Later it was Maxwell who had the acuity and perception to first see the use of energy or the partition of energy as a possible failure criterion. In correspondence with Lord Kelvin in 1856 Maxwell anticipated what is now known as the Mises criterion by saying "I have strong reasons for believing that when the strain energy of distortion reaches a certain limit then the element will begin to give way," Timoshenko [3]. Much later, Huber [4] and still later Mises [5] gave this the form in which it is used today for the yielding of ductile metals, but not for anything more general than that.

The forms of the Mises and Tresca failure criteria for three dimensional stress conditions in terms of principal stresses are given by

$$\text{Mises} \quad \frac{1}{6} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \leq S^2 \quad (2)$$

and

$$\text{Tresca} \quad \frac{1}{2}(\sigma_1 - \sigma_3) \leq S \quad (3)$$

where in the case of the Tresca criterion  $\sigma_1$  is the maximum principal stress and  $\sigma_3$  is the minimum and  $S$  is the yield or failure stress in shear. In the Mises criterion the order of the stress components does not matter. The maximum difference between the two criteria is 14.4%.

Over the years the appeal of various forms of energy as a general failure criterion has been elusively strong. Beltrami [6] proposed that total energy, the energy of distortion plus the energy of volume change, be used as the general failure criterion. But that is clearly contradicted by the ductile metals case and many other materials types as well.

In addition to other approaches, the maximum normal stress and maximum normal strain failure criteria have had distinguished and strong adherents – Rankine and Lamé for the former and Saint-Venant and Poncelet for the latter, Timoshenko[3]. Neither came into a state of explicit validation and meaningful application, although they certainly added to the

historical state of confusion. These two failure criteria are sometimes still used, with no apparent general justification.

In the early 1900's Mohr [7] returned to Coulomb's original failure form (1). Mohr gave an interpretation of Coulomb's two parameter form by using the maximum and minimum principal stresses to form a "Mohr's circle" construction inside the linear envelopes of (1) as shown in Fig. 1.

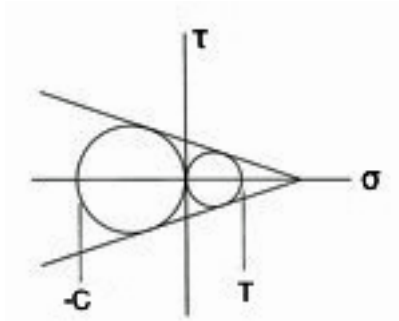


Fig. 1 Coulomb-Mohr failure criterion

Incipient failure occurs when the circles have tangency with the linear envelopes. This then is called the Coulomb-Mohr or the Mohr-Coulomb criterion. This usually is left in different analytical forms, mostly quite involved ones, but they all reduce to the incredibly simple form of

$$\frac{\sigma_1}{T} - \frac{\sigma_3}{C} \leq 1 \quad (4)$$

where T is the failure stress in uniaxial tension and C is that in uniaxial compression, and  $\sigma_1$  is the maximum principal stress and  $\sigma_3$  is the minimum. In principal stress space the Coulomb-Mohr failure criterion is represented as a six-sided pyramid, coming to a sharp vertex and opening indefinitely in the opposite direction.

Thus the Mises and Tresca criteria are one parameter forms and the Coulomb-Mohr criterion is of a two parameter form which would have more generality than could a one parameter form. After its development there was high promise and even excitement for the Coulomb-Mohr failure theory. So much for short term promise. In the 1920's von Karman [8] and Böker [9] convincingly showed that for geological materials the Coulomb-Mohr

criterion is only partially successful when the stresses are somewhat compressive and not at all successful otherwise. Voigt also found that the Coulomb-Mohr form could not model general failure behavior, but he was so negative about its prospects that he left his important research unpublished, Timoshenko [3]. The Coulomb-Mohr criterion gradually faded into the background as being only a historical artifact, at least insofar as research is concerned. It completely failed to demonstrate relevance for a wide range of isotropic materials types.

Perhaps because of the disappointment in the Coulomb-Mohr failure criterion Drucker and Prager [10] in the 1950's introduced another two parameter failure form. It is represented by a conical surface in principal stress space rather than as a pyramid, but it has a rather complicated analytical form. Unfortunately it has not been any more successful as a general failure criterion than the Coulomb-Mohr form, both are unsatisfactory.

After the many attempts to cast the Coulomb-Mohr hypothesis into a generally realistic, accessible form never came to fruition, and the quest for a general energy criterion necessarily came to an unsuccessful end, a degree of pessimism seemed to influence the further efforts to find a general criterion. Thereafter, the attempts were direct postulations of particular forms, appealing on some basis to each originator, but apparently done with a motivation to just see what would happen. None of the general approaches in the modern era had a solid basis in a physical derivation along with a critical examination, there have only been demonstrations. For some examples see the survey by Paul [11].

The historical status of materials failure theory right up to the near present isn't so much different than it was in the time of Benjamin Franklin. Franklin once famously said "Nor is it [of] much importance to us to know the manner in which nature executes her laws, it is enough if we know the laws themselves. It is of real use to know that china left in the air unsupported will fall and break, but how it comes to fall and why it breaks are matters of speculation. It is a pleasure indeed to know them, but we can preserve our china without it.", Isaacson [12]. Franklin gravely underestimated Isaac Newton's magnificent contribution but he had it about right on the tea cup (china) breakage problem. Benjamin Franklin was an extremely acute and perceptive observer of the physical and electro-magnetic world around him. Had he the benefit of a physics and

mathematics education he might well have been an all time great scientist, and who knows, he might have helped with the materials failure problem too just as Coulomb tried to do.

The field of materials failure has been mysteriously held up in the “air” for centuries but no one quite knows how or why.

### 3. The Pervasive and Enduring Dilemma



The on and off consideration of the Coulomb-Mohr criterion had extended for over more than a hundred years and still it had not produced the desired result. The newer phase of the search centered around a seeming consensus that two parameters in the failure criterion could not do the job and more, perhaps many more parameters would be needed. A consequence of the many parameters approach was that the emphasis would be given to particular classes of materials with no expectation or possibility of generality. Some would be for the different classes of metals, some for the different classes of polymers and so on. Such empirical forms could have some utility but each would have undefined and vague limits of applicability. Essentially they could only be used for minor interpolation purposes.

The one parameter Mises and Tresca criteria are only applicable to ductile metals. For this application the Mises criterion is the preferred and well justified form, while the Tresca form is only an approximation to it. For anything but this application to ductile metals these two failure criteria can make ridiculous predictions. One would think that the two parameter criteria would be better but they too can be ridiculous, as shown next.

The complete unsuitability of the Coulomb-Mohr and the Drucker-Prager two parameter forms are easily verified. In the case of the Drucker-Prager criterion if  $T/C \leq 1/3$ , the failure criterion predicts that a stress state of

$$\begin{aligned}\sigma_1 &= \sigma_2 = -\sigma \\ \sigma_3 &= 0\end{aligned}\tag{5}$$

allows unlimited compressive stresses. T and C are the uniaxial tensile and compressive strengths. Thus a material such as cast iron would be claimed to support unlimited stress magnitudes in eqi-biaxial compression. This is of course impossible.

In the case of the Coulomb-Mohr criterion a stress state of

$$\begin{aligned}\sigma_1 &= \sigma_2 = -\sigma \\ \sigma_3 &= -2\sigma\end{aligned}\tag{6}$$

is claimed to support unlimited compressive stresses for all brittle materials with  $T/C \leq 1/2$ . This also is a physically impossible prediction because of the distortional stress state in the material.

These four failure criteria have absolutely no hope of general applicability. Unfortunately the community of technical users of failure criteria are lead to believe otherwise. Any or even most text books on materials and on mechanics state the Mises and Tresca criteria as the standard and accepted forms. There is no warning label on their inapplicability or even a cautionary note. At a more advanced level technical people are aware of or at least have heard of the Coulomb-Mohr and the Drucker-Prager criteria and they have a certain appeal based upon the tremendous prominence of their names. These criteria are often used purely on this implied status of their progenitors. This understandable but completely misplaced confidence can lead to drastically incorrect predictions of safety. All of these forms have the superficial appearance of applicability and generality when in actuality they have no such thing. This is the unavoidable dilemma that has accrued from a history of misdirection.

The present situation for materials failure prediction is pernicious. This state of mis-information is propagated in the classrooms and in the text books. There is an urgent need for educational reform. But here is where a seeming paradox arises. How can the entrenched norms of the teaching tools for failure be rectified and reformed unless and until they can be replaced by something more substantial and completely reliable? Where will that come from?

Certainly the answer is not with the development of further empirical failure criteria, usually involving three or more materials parameters to be varied to fit any particular set of testing data. This would be called the many



parameters approach. It is very unlikely to ever succeed. Consider the following. If the parameters approach were applied to three-dimensional elastic behavior, rather than using the classical theoretical foundation of elasticity theory, the result would certainly be totally useless and absurd. There is no reason to believe that the many parameters approach would be any more successful for failure characterization.

After all this time and all these attempted developments, clearly the state of general three-dimensional failure characterization for homogeneous and isotropic materials is completely unsatisfactory. There have literally been hundreds or perhaps thousands of failure forms displayed and proposed for use but still there is no generality, only empiricism.

Again it can be asked, is there any hope? Despite the long, difficult, and unpromising history, there remains the possibility that the expanding knowledge base of the modern era may provide the advantage needed for a more successful formulation of materials failure theory.

#### **4. A Glimmer of Light**



If one is to set out to develop a completely general and physically realistic theory of failure, where will the inspiration come from to get started? One source has already been mentioned, the step by step development of fracture mechanics. This remarkable development occurred over a period of 30 or 40 years. Another inspiration would be Coulomb's recognition of the first known failure criterion, (1). In effect it says that the level of shear stress at failure must depend upon the state of hydrostatic stress in the material. That much is certainly true. Unfortunately Mohr's exploitation of (1), promising though it seemed at first, turned out to be a dead end. As far as inspiration is concerned, that's one huge success and one total failure (failure of purpose).

Probably the word inspiration is not even the right term for the intended purpose. The correct term must convey the crucial step(s) needed to get started. There needs to be a recognition of some new coordination of physical effects/behaviors, something that has always been there in the mass

of disjointed information on failure but never before recognized and never before utilized. There must be a special association of all the loose facets and facts of failure behavior, assembled in such a way to amount to more than just the sum of the parts. In fact, it turns out that there are two such major turning-points that will completely enable a new theory of failure. They are definitely glimmers of light, perhaps incandescent light.

### The Organizing Principle

When one assembles a table of the uniaxial tensile and compressive strengths for all the different classes of homogeneous and isotropic materials it at first seems like nothing so much as a vast conglomeration of numbers with a very wide range of magnitudes and nothing more. But with further inspection, an interesting and revealing pattern begins to emerge from the mass of T and C strength data. The unifying and organizing basis for materials failure becomes apparent when the spectrum of the T/C ratio values are formed. The T/C strengths ratio captures the entire spread from brittle behavior at T/C=0 to ductile behavior at T/C=1 and covering the entire spectrum in between as

$$0 \leq \frac{T}{C} \leq 1 \quad (7)$$

This brittle to ductile change with the variation of the uniaxial tensile to compressive strengths ratios T/C's is the organizing principle for materials failure.

Letting stress be nondimensionalized by C as

$$\hat{\sigma}_{ij} = \frac{\sigma_{ij}}{C} \quad (8)$$

then suggests that the entire theory of failure in terms of nondimensional stress may depend only upon the T/C materials strength ratio. This would leave the complete theory dependent upon only on the two failure properties, T and C.

This organizing principle does not by itself determine the theory of failure but it resolutely opens the door to pursue an approach to a two property failure theory. The table of T/C values for the various materials

classes and types will be shown in the next section of this account. First though, consider the second turning-point.

### The Critical Relationship Between Elasticity and Failure

To start at the beginning, first the theory of the elastic behavior of materials must be derived. Then one can take the hypothesis that failure effectively represents the cessation of the capability of the material to store elastic energy. Simple though this may seem, it actually is far more subtle than it appears. The obvious next step would be to take the termination of strain energy at some value as the failure criterion. That simplistic approach is the historical trap that keeps being rediscovered only to later find that it is blatantly incorrect.

The correct way to pursue the concept of failure as the termination of the capacity to store energy is as follows. After deriving the linear elastic energy, stress-strain constitutive relations, attention is next turned to the constitutive relation for the failure of the elastic material. The failure type constitutive relation would be expected to be related to but formally independent of the elastic energy, stress-strain forms. The same formalism as used to find the linear elastic constitutive relation for energy can then be used to independently derive the constitutive relation for failure.

This approach cannot as yet be considered to be a method, but like the preceding organizing principle and in coordination with it, it opens the door to develop the method based upon these two hypotheses. It will become the vital turn-key operation leading to the failure theory development.

There is one obstacle to this approach that must be recognized, rationalized and surmounted. Except in the case of perfectly brittle fracture, failure has always implicitly and explicitly been taken to be the terminus of the plastic deformation after the yielding and strain hardening of the material transpires. This view has been the dominating consensus for at least the past one hundred years. In contrast, it is here said that failure represents the termination of the elastic capability to store energy. These two conflicting views must be reconciled in order to proceed. At first they may seem incompatible but actually the proper perspective is as follows. The failure state absolutely does represent the termination of the elastic deformation capability but when plastic behavior occurs it simply represents a more complex transition and path from the elastic state to the failure state. It does

not change the fundamental character of what failure most basically represents. Elasticity and failure are deeply intertwined and interrelated.

These two new hypotheses will enable the complete development of the theory of materials failure, to be stated next.

## 5. One Book and Six Papers



Using the insight and opportunity provided by the two hypotheses of the previous section, the complete and comprehensive theory of materials failure has been derived, not postulated. The derivation is presented in the recent book

The Theory of Materials Failure (2013), Oxford University Press, Oxford, U. K.

After the publication of the book, six adjoining papers on materials failure were written and published as follows.

1. “Failure Mechanics – Part I: The Coordination Between Elasticity Theory and Failure Theory for all Isotropic Materials,” (2014), Journal of Applied Mechanics, 81, 081001-1.

2. “Failure Mechanics – Part II: The Central and Decisive Role of Graphene in Defining the Elastic and Failure Properties for all Isotropic Materials,” (2014), Journal of Applied Mechanics, 81, 111001-1.
3. “Failure Mechanics – Part III: A Call to Service With Solid Mechanics,” (2015), Journal of Applied Mechanics, 82, 041001-1.
4. “A New Theory of Strain Hardening and its Consequences for Yield Stress and Failure Stress,” (2015), Computers, Materials and Continua, 47, 45-63.
5. “Evaluation of Ductile/Brittle Failure Theory and Derivation of the Ductile/Brittle Transition Temperature,” (2016), Journal of Applied Mechanics, 83, 022001-1.
6. “The Theoretical Measure of the Ductility of Failure for All Isotropic Materials in All States of Stress,” (2016), Journal of Applied Mechanics, 83, 061001-1.

The titles of the papers are mostly self explanatory and the papers need not be abstracted here. Paper No. 3 concerns the crisis in the teaching of materials failure.

These papers do not provide background for the book nor do they provide supplements for it, the book is self contained and free standing. The papers are also free standing and involve further research directions on the subject of materials failure. However, the papers do complement the book in the following sense. The book is at quite high level, it is not casual reading on the topic. The papers are at an even higher level, probing research areas opened up by the advent of the book. The papers add to the significance of the book and fortify its relevance and its methodology for treating materials failure.

Taken together, the book and the six papers comprise a thorough and far reaching treatment of materials failure. A few sample but salient points from the book and some of the papers will be outlined next. These should be enough to give some indication of their contents. Further interests must be referred to the book and the papers themselves. Oxford University Press

made the book available as a paperback in early 2017. The papers are available directly from ASME.

The derived failure criterion is actually two separate and competitive criteria. They will now be stated. They apply to homogeneous and isotropic materials.

### Polynomial Invariants Failure Criterion

For the complete range of T/C's given by (7) the compact form of the first failure criterion is

$$\left(1 - \frac{T}{C}\right) \hat{\sigma}_{ii} + \frac{3}{2} \hat{s}_{ij} \hat{s}_{ij} \leq \frac{T}{C} \quad (9)$$

where  $\hat{\sigma}_{ii}$  is from the nondimensional dilatational stress and  $\hat{s}_{ij}$  is the nondimensionalized deviatoric stress.

### Fracture Criterion

This second failure criterion applies over the partial range of T/C's as shown by

$$\text{For } \frac{T}{C} \leq \frac{1}{2} \quad \hat{\sigma}_1 \leq \frac{T}{C} \quad (10)$$

where  $\hat{\sigma}_1$  is the nondimensionalized maximum principal stress.

Both (9) and (10) must be evaluated to find which one controls for any given stress state.

In principal stress space (9) is a paraboloidal surface. The fracture criterion (10) cuts three planar surfaces off from the paraboloidal envelope. Even though there are two competitive failure criteria the resulting failure envelope is always continuous.

The fundamental role played by the uniaxial strengths ratio T/C is obvious in (9) and (10). The value of T/C is designated as the materials type. The almost unbelievably broad materials classes of relevance here are:

Ductile and brittle metals  
Ductile and brittle polymers  
Ceramics  
Glasses  
Isotropic geological materials  
Other specialized classes such as amorphous metals

The values of the T/C materials designation for these various materials classes can overlap with each other.

The role of T/C is best understood from a table of the T/C values for the various materials classes. Such a table is shown below revealing how the T/C values span the whole range of ductile versus brittle failure characteristics.

<b>Materials Type</b>	<b>T/C</b>	<b>Predicted D/B Behavior in Uniaxial Tension</b>
Aluminum	1	Perfectly Ductile
Steel	1	Perfectly Ductile
Polyethylene	0.9	Extremely Ductile
Polycarbonate	0.8	Very Ductile
Epoxy	2/3	Ductile
Nickel & Polystyrene	1/2	D/B Transition
Cast Iron	1/3	Brittle
Silicon Carbide	1/5	Very Brittle
Float Glass	1/10	Extremely Brittle
Dolomite	1/15	Extremely Brittle
Some Geological Materials	1/50 to 1/100	Extremely or Totally Brittle

Table 1 T/C and the ductility scale in uniaxial tension

In developing a full and comprehensive failure theory, the failure criteria are but part of the proceedings. The other equally important part is and must be an understanding of and the systematic development of all matters related to ductile versus brittle failure, including the ductile/brittle transition.

The full development of all ductile/brittle matters are given in the book and in some of the papers. One of the main developments is the



derivation of a method for giving a quantitative measure of the ductility of failure for any materials type through its T/C value and for any stress state. The end result is the definition of the failure number,  $F_n$ , given by

$$F_n = \frac{1}{2} \left( 3 \frac{T}{C} - \hat{\sigma}_{ii}^f \right) \quad (11)$$

where  $\hat{\sigma}_{ii}^f$  comes from the failure level critical stresses in either of the failure criteria (9) or 10).  $F_n=0$  is no ductility and  $F_n=1$  is perfect ductility. If in the definition (11) for a particular stress state it is found that  $F_n < 0$  then it reverts to 0 and if  $F_n > 1$  then it reverts to 1, the respective cases of no ductility and perfect ductility.

For example, for a state of uniaxial tension  $\sigma_{11}=T$  and then  $\hat{\sigma}_{ii}^f=T/C$  and that into (11) gives

$$F_n = \frac{T}{C} \quad \text{Uniaxial Tension} \quad (12)$$

With (12) it is now seen that Table 1 represents the quantitative values of the ductility in the state of uniaxial tension. The same method gives the quantitative ductility in any stress state.  $F_n=1/2$  designates the ductile/brittle transition for any stress state.

The subject of materials failure is extremely broad and inclusive. Developing a theory to cope with these conditions is an extraordinarily demanding objective. Even further, to imagine that this could be accomplished with calibration from only two basic strength properties would seem to border on the impossible. Yet it has been done, evaluated, verified, and documented. This book and these six papers are the result.

None of this would have been possible were it not for the bedrock organizations and traditions of science and engineering. There are no finer outlets for original research than Oxford University Press and Journal of Applied Mechanics. The Journal of Applied Mechanics came into being when Stephen Timoshenko founded the Applied Mechanics Division. A tradition of distinguished editors followed, right up through the current editor, Yonggang Huang. Journal of Applied Mechanics was the

first separate ASME journal, beyond the Transactions, and it remains and continues as the flagship journal of the society. There could not be higher professional goals and aspirations for anyone than the advancement of science and engineering as brought into effect through these institutions.

## 6. Where Do We Go From Here



What a ragged history this important and vital field has had. It has gone from a state of falsely based high elation to apathy and even despair. After the “failure” of the Coulomb-Mohr approach, the field gradually sank into the condition of resigned, curve fitting empiricism, and there it has steadfastly remained.

There always was the hope that someone would have a brilliant idea, an out of the blue inspiration. That never happened and for good reason. It would require more than just that to formulate a consistent theory of materials failure. The situation was much like that in the very early days of the development of elasticity theory. There was uncertainty and great controversy as to whether isotropic elasticity theory was a one constant theory or a two constant theory. The two constant (two properties) form eventually won the controversy because of its thoughtful, careful, rational formulation, later to be verified experimentally. No less than that was and is required for a successful approach to materials failure theory.

Now that there finally is a comprehensive theory of materials failure that has been evaluated and verified, it is the opportune time to move on to the next priorities. One next step will involve building up a catalog of applications of the failure theory. Concurrently something must be done to reform the educational/tutorial approach to teaching the subject of materials failure. It is in a dreadful state of disrepair, see Paper 3 in the Section 5 list of six papers. The current teaching status constitutes a roadblock that must be cleared before further progress can occur.

When will change come, 1 year, 10 years, more; it is unpredictable. Right now, year 2018, is when the clock starts ticking on this. Such drastic course corrections do not come easily or naturally or

quickly. Careers and professional commitments usually must accommodate and conform to the status quo or at most allow only small deviations from it. This is especially true for the books and the tutorial tools that propagate the “old” teaching of materials failure. Despite that, ultimately the pressure will increase until it finally reaches an unstable limit, not unlike that of approaching materials failure itself in testing. Then there will be a sudden change, an accelerated effort and program for the adoption of the rational treatment of materials failure. It will come, that much is certain, only the time scale is uncertain. There is little doubt that materials science and engineering and mechanical engineering and civil engineering will be leading the charge, it has already begun with “one book and six papers”.

## 7. Historical References

1. McCullough, D. (2015), The Wright Brothers, Simon and Schuster, New York.
2. Coulomb, C. A. (1773), In Memories de Mathematique et de Physique, Academie Royal des Siences par divers sans, 7, 343-382.
3. Timoshenko, S. P. (1953), History of Strength of Materials, McGraw-Hill, New York.
4. Huber, M. T. (1904), “Przyczynek do Podstaw Wytorymabsci,” Czapismo Technizne, 15, 81.
5. von Mises, R. (1913), “Mechanik des festen Korper im Plastisch-Deformablen Zustand,” Nachr. Kgl. Ges. Wiss. Gotingen Math.-Physik Klasse, 582-592.
6. Beltrami, E. (1889), “Considerazione Idrodinamiche,” Rend. 1 Lombardo Sci. Lettere, 22, 121-130.
7. Mohr. O. (1900), “Welche Umstande Bedingen die Elastizitatsgrenze und den Bruch eins Materials,” Zeitschrift des Vereines Deutscher Ingenieure, 44, 1524-1530.
8. von Karman, T. V. (1912), “Festigkeitversuche unter allseitigem

Druck,” Mitt. Forschungsarbeit Gebiete Ingenieures, 118, 27-68.

9. Böker, R. (1915), “Die Mechanik der Bleibenden Formänderung in Kristallinisch Aufgebauten Körpern,” Mitteilungen Forschungsarbeit auf dem Gebiete Ingenieurwesens, 24, 1-51.
10. Drucker, D. C., and Prager, W. (1952), “Soil Mechanics and Plastic Analysis or Limit Design,” Quart. Appl. Math., 10, 157-165.
11. Paul, B. (1968), “Macroscopic Criteria for Plastic Flow and Brittle Fracture,” in Fracture (ed H. Liebowitz), II, 313-496, Academic Press, New York.
12. Isaacson, W. (2003), Benjamin Franklin, Simon and Schuster, New York.

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