XXII. LAMINATION THEORY FOR THE STRENGTH OF FIBER COMPOSITE MATERIALS

Introduction

The lamination theory for the elastic stiffness of fiber composite materials is the backbone of the entire field, it holds it all together. A corresponding and related lamination theory for the strength of laminates would be equally important and useful. It would complete the understanding of the behavior of fiber composite materials in its most important and prime form of usage, laminates.

This research report extends the technical program established in Ref. [1]. In that work Christenen and Lonkar [1] developed failure criteria that were mainly intended for quasi-isotropic fiber composite laminates. This work develops the much broader and much more generally useful failure criteria for wide classes of orthotropic fiber composite laminates.

The quasi-isotropic laminates case is important because it is one limiting case of fiber composite laminations. The other limiting case is the unidirectional form itself. The class of orthotropy supplies the general and inclusive bridge of cases in between and including these two limiting cases. General cases such as these have been treated in the large and highly publicized composites failure program of Hinton and Kaddour [2]. The end result of that program served more to show the utter and unrelenting complexity of the overall problem rather than providing any unified and directly applicable results.

The following work will begin by recalling some previous results from Ref. [1] and then pursuing a very large scale extension of the method. This will allow the treatment of the general case of orthotropic laminates where the degree of anisotropy in the plane of the laminate can be arbitrarily specified as necessary for a very wide range of layup patterns and problem specifications.

At the lamina level the fiber composites form will be taken as highly anisotropic, appropriate to very high stiffness and high strength fibers (typified by carbon fibers) in a polymeric matrix. It will be found that the failure criteria for the general orthotropic laminate is completely calibrated by only two fiber dominated failure properties at the unidirectional lamina or the tow level of the fiber-matrix system. Calibrating the failure theory for general orthotropic laminates by use of only two failure properties certainly would have been thought to be impossible. This seemingly intractable problem will be solved here, removing a large obstacle to meaningful fiber composites applications.

The case of unidirectional fiber composites failure was treated in Ref. [3]. As already discussed, the complementary case of the failure of quasi-isotropic laminates was treated in Ref. [1]. Associated with these two works, then the present work completes the longstanding task by rigorously treating the failure of a very wide class of orthotropic laminates.

This new work on composites failure could equally well be presented in a mechanics oriented materials journal or a composite materials journal. The application is to composites but the methodology is totally mechanics based. This development would not be possible were it not for the insight and avenues of approach opened up by only the mechanics discipline. The usual composites failure approach zeroes in on one particular failure mechanism and treats that in some, usually great detail. The mechanics approach used here is much broader and more inclusive, but not at the expense of rigor. The present approach will be found to be satisfyingly rigorous and general

The primary reason for conducting this research and writing this paper is to provide a viable tool for use with computational mechanics. Composites research sources seem to be pre-occupied with other matters and not interested in pursuing the general failure problem. Perhaps the failure problem is considered by them to be too difficult or even impossibly difficult. Nevertheless the problem desperately needs attention, solution, and subsequent application. This work accepts the challenge to produce general failure results so that computational mechanics can then take over and open up new directions for the applications of fiber composite materials.

Orthotropic Laminates Failure Theory

In Ref. [1] Christensen and Lonkar derived the failure criterion for quasi-isotropic laminates. It was shown to have the same form as that for three-dimensionally isotropic materials when in the 2-D state of plane stress. It is given by

$$\left(\frac{1}{T}-\frac{1}{C}\right)\sigma_{11}+\left(\frac{1}{T}-\frac{1}{C}\right)\sigma_{22}+\frac{1}{TC}\left[\sigma_{11}^2-\sigma_{11}\sigma_{22}+\sigma_{22}^2\right]+\frac{3}{TC}\sigma_{12}^2\leq 1 \qquad (1)$$

where T and C are the uniaxial strengths of the laminate. Now rewrite this in slightly modified but equivalent form that will be helpful in the following orthotropic developments, as given by

$$\left(\frac{1}{T}-\frac{1}{C}\right)\sigma_{11}+\left(\frac{1}{T}-\frac{1}{C}\right)\sigma_{22}+\left[\left(\frac{\sigma_{11}}{\sqrt{TC}}\right)^2-\left(\frac{\sigma_{11}}{\sqrt{TC}}\right)\left(\frac{\sigma_{22}}{\sqrt{TC}}\right)+\left(\frac{\sigma_{22}}{\sqrt{TC}}\right)^2\right]+\frac{3}{TC}\sigma_{12}^2\leq 1 \tag{2}$$

In Ref. [1] the quasi-isotropic form in (2) was shown to generalize to the failure criterion for orthotropic laminates in plane stress as

$$\begin{aligned}&\left(\frac{1}{T_{11}}-\frac{1}{C_{11}}\right)\sigma_{11}+\left(\frac{1}{T_{22}}-\frac{1}{C_{22}}\right)\sigma_{22}+\\&\left[\left(\frac{\sigma_{11}}{\sqrt{T_{11}C_{11}}}\right)^2-\left(\frac{\sigma_{11}}{\sqrt{T_{11}C_{11}}}\right)\left(\frac{\sigma_{22}}{\sqrt{T_{22}C_{22}}}\right)+\left(\frac{\sigma_{22}}{\sqrt{T_{22}C_{22}}}\right)^2\right]+\frac{\sigma_{12}^2}{S_{12}^2}\leq 1\end{aligned} \tag{3}$$

where $T_{11}, C_{11}, T_{22}, C_{22}$ are the tensile and compressive strengths in the 1 and 2 stress directions and S_{12} is the shear strength. The orthotropic form given in Ref. [1] appears a little different from that in (3) but it is equivalent.

These five calibrating strengths are directly measurable from standard, single stress component 1-D tests. Form (3) is remarkably simple for the failure criterion of fully and generally orthotropic laminates. It will be developed even much further in the following derivation.

For these further developments, temporarily return to the case of the quasi-isotropic laminate, as treated in Ref. [1]. The failure criterion (1) only requires two strength properties to completely calibrate it. These are the uniaxial tensile and compressive strengths T and C. In a special development in Ref. [1] it was shown that the uniaxial strengths T and C for the laminate can be expressed in terms of the unidirectional fiber composites tow strengths as

$$T=\frac{T_{tow}}{3} \tag{4}$$

and

$$C = \frac{C_{tow}}{3} \tag{5}$$

These are physically realistic approximations that apply for fiber dominated systems. The term "fiber dominated" means that the fibers are greatly stiffer and stronger than is the surrounding polymeric matrix phase of the composite material. In effect then the fiber phase is a completely one dimensional reinforcing agent, meaning that the fiber contribution to the stiffness and strength of the laminate is purely through its unidirectional, axial direction contribution.

A slightly more complicated quasi-isotropic form than (5) for the compressive strength property was derived in Ref. [1]. The simpler form (5) will be used here. It is here considered that the form in Ref. [1] is more appropriate for use with the unidirectional compressive strength being obtained from a very thin lamina form allowing the kink band mechanism of failure whereas the form (5) is more appropriate for use when the unidirectional compressive strength as determined from a monolithic type test specimen. The form (5) will be used here as being representative of the compressive strength obtained directly from the impregnated fiber tow as manufactured rather than from the further processed lamina form.

Next the fiber dominated forms (4) and (5) for the quasi-isotropic case will be generalized for use with the orthotropic case. This will be found to be a major and completing step forward in understanding general composites laminate strength behavior.

The orthotropic laminate will be taken as the standard 0, 90, ± 45 degree layup pattern as shown in Fig. 1.

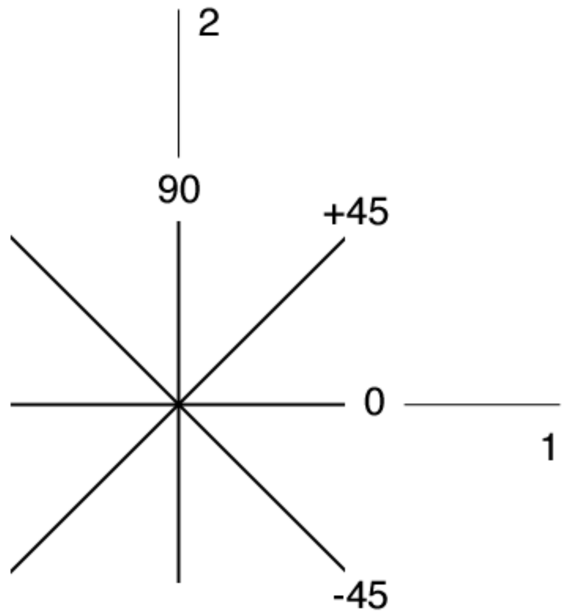

Fig. 1 Lamination pattern

Take $c_0, c_{45}, c_{-45}, and\ c_{90}$ as the specified volume fractions of the four lamina orientations in the orthotropic laminate. For orthotropic symmetry it is required that

$$c_{-45} = c_{45} \tag{6}$$

Thus it follows that

$$c_0 + 2c_{45} + c_{90} = 1 \tag{7}$$

The objective here is the generalization for fiber dominated systems from the quasi-isotropic case (4) and (5) to the orthotropic case. The five calibrating strengths $T_{11}, C_{11}, T_{22}, C_{22}, and\ S_{12}$ will now be derived in terms of the general volume fractions $c_0, c_{90}, and\ c_{45}$ and they will be left open to later specification for particular cases.

First consider the shear strength S_{12}. For the fiber dominated system the 0 and 90 plies make no contribution to the shear strength, only the 45 plies give shear resistance. Furthermore the shear strength must be proportional to the volume of the active material in shear deformation and thus must be linearly dependent on c_{45}. Then take

$$S_{12} = \alpha c_{45} \sqrt{T_{tow} C_{tow}} \tag{8}$$

where α is an unknown coefficient to be determined. The dependence shown in (8) on T_{tow} *and* C_{tow} follows from the quasi-isotropic form for S from (1) as

$$S = \sqrt{\frac{TC}{3}} = \frac{1}{3}\sqrt{\frac{T_{tow} C_{tow}}{3}} \tag{9}$$

Lastly parameter α in (8) at c_{45} = ¼ then requires

$$\alpha = \frac{4}{3\sqrt{3}} \tag{10}$$

to recover the form in (9). This completes the determination of orthotropic S_{12}. The other 4 cases for the uniaxial strengths are considerably more complex to determine.

Before proceeding to the next cases it should be noted that this fiber dominated approach can give some strength properties as being zero. For example, if there were no ± 45 plies then the shear strength (8) would vanish. Of course this just would mean that the failure is totally matrix controlled and would be of very low strength capability compared with the fiber dominated and controlled failure modes. This would further mean that this would be a very poor design and should be discarded.

The path to be followed for the determination of $T_{11}, C_{11}, T_{22}, C_{22}$ is exactly like that of the classical lamination theory used with stiffness. But instead of stiffness, strength will be the determining factor. With stiffness the lamination theory procedure usually requires a computer to keep account of the bookkeeping for the tensor transformations. But for the fiber dominated strength problem the procedure is much simpler since the fibers are one dimensional load bearing members. In fact it is rather intriguing to watch the theory unfold and lead to surprisingly simple and clear results.

First for the determination of the uniaxial strength T_{11} take a unit strain in the T_{11} direction. This induces a strain and consequently a fiber direction stress in the ± 45

lamina, for these fiber dominated lamina. Now resolve this c_{45} stress into the coordinate axes directions using the tensor transformations. For no transverse stress in the laminate to exist, the 2 direction stress in the ± 45 lamina must be resisted by the 90 degree lamina. This leads to the determination of the orthotropic Poisson's ratio ν_{12} as

$$\nu_{12} = \frac{1}{1 + 2\dfrac{c_{90}}{c_{45}}} \tag{11}$$

Similarly it is found that

$$\nu_{21} = \frac{1}{1 + 2\dfrac{c_0}{c_{45}}} \tag{12}$$

When $c_{45} = 0$ then from (11) and (12) $\nu_{12} = \nu_{21} = 0$ as must be the case for the fiber dominated system in a cross-ply laminate. When $c_{90} = 0$ in (11) then $\nu_{12} = 1$ meaning that there is no resistance to lateral strain change. Similarly when $c_0 = 0$ in (12).

Finally the resolution of all the fiber direction stresses into the 1 direction gives the uniaxial tensile strength in that direction as

$$T_{11} = \left(c_0 + \frac{c_{45}c_{90}}{c_{45} + 2c_{90}} \right) T_{tow} \tag{13}$$

Similarly it can be derived that

$$C_{11} = \left(c_0 + \frac{c_{45}c_{90}}{c_{45} + 2c_{90}} \right) C_{tow} \tag{14}$$

and

$$T_{22} = \left(c_{90} + \frac{c_{45}c_0}{c_{45} + 2c_0} \right) T_{tow} \tag{15}$$

$$C_{22}=\left(c_{90}+\frac{c_{45}c_0}{c_{45}+2c_0}\right)C_{tow} \tag{16}$$

The completing shear strength from (8) and (10) is given by

$$S_{12}=\frac{4c_{45}}{3}\sqrt{\frac{T_{tow}C_{tow}}{3}} \tag{17}$$

Relations (13)-(17) are the full strength properties needed to calibrate the general orthotropic failure criterion (3). They also are of great interest in their own right for an orthotropic laminate.

The uniaxial strengths (13) - (16) can also be written in terms of the Poisson's ratios in the compact forms

$$T_{11}=\left(c_0+v_{12}c_{90}\right)T_{tow} \tag{18}$$

$$C_{11}=\left(c_0+v_{12}c_{90}\right)C_{tow} \tag{19}$$

$$T_{22}=\left(c_{90}+v_{21}c_0\right)T_{tow} \tag{20}$$

$$C_{22}=\left(c_{90}+v_{21}c_0\right)C_{tow} \tag{21}$$

where $v_{12}\ and\ v_{21}$ are given by (11) and (12).

The basic relations (3) and (13) – (17) are powerful results. Not only do they represent the solution to a previously unsolved, very important problem, they do so in an exceedingly simple, clear and concise form. They comprise the orthotropic lamination theory for the strength of fiber dominated composite materials. Contrary to the author's initial and cursory expectation, this much more complex problem and solution is fully as rigorous as the specialized quasi-isotropic results in [1]. They include the quasi-isotropic forms as a special case.

The uniaxial and shear strengths (13) – (17) satisfy all the necessary consistency tests. These are as follows. First note that for a unidirectional lamina (13) gives $T_{11}=T_{tow}$ as must occur. The strengths (13) – (17) also have the necessary forms for a fiber dominated cross-ply laminate necessarily having no shear resistence. The complementary case of only ± 45's shows the "scissoring effect with no resistance that must occur. Finally, when relations (13) – (17) are reduced to the quasi-isotropic case by taking $c_0 = c_{90} = c_{45} = 1/4$ the correct quasi-isotropic results are recovered from (13) – (17), namely (4) and (5).

This fiber dominated orthotropic failure theory can be derived for any orthotropic laminate, not just for 0, 90, ± 45 laminates. The forms are a little more complex, but entirely manageable. Such results will be presented in due course. But the 0, 90, ± 45 case is the most important and the simplest case and the one that completely and naturally conforms to orthotropic symmetry. It is emphasized here.

The five strengths in (13) – (17) are entirely specified by only the two tow strengths for any orthotropic, 0, 90, ± 45 layup. Of course this result is restricted to fiber dominated systems such as carbon fiber, polymeric matrix composites.

The enabling operation for this unexpected development is that these five calibrating properties for the orthotropic case are only one dimensional problems. In these cases one can reason which lamina orientations in the four directions are applicable for fiber dominated systems and which are inapplicable for each of the five properties. This operation could not be accomplished if it involved biaxial stress conditions. Effectively critical strain in the 1-D fiber direction specifies the failure condition in the uniaxial cases aligned with the symmetry axes.

It should be emphasized that fiber dominated systems cannot sustain all stress states. For example for $c_{\pm 45} = 0$ then (13) – (15) cannot support a shear stress state. But this simply means that the matrix phase controls failure for such a condition. That would be of no interest for fiber composites. The fiber dominated state must be designed to support the load under all anticipated loading conditions.

The final and complete statement of failure criteria for orthotropic laminates is given by the failure criterion (3) with the five calibrating strengths specified by (13) – (17) as expressed in terms of the two tow strength properties. To imagine that the complete strength specification for any stress state in the orthotropic laminate could be specified from only two experimental measurements would have seemed impossible before the derivation in this paper.

A Specific Example and General Failure Results

Everything up to this point up to this point has been motivated and necessitated by original research. Now some interesting consequences from the general theory will be developed and then used in an example.

Full blown design procedures for failure would normally involve investigating many different states of stress that could occur in practice. The simplest possible problem of this type will be opened here by prescribing the allowable stress states for the two uniaxial tensile stresses and for the shear stress, taken as individual conditions. After that the more general case will be taken up.

As a simple but realistic example take the volume fractions of the lamina orientations as

$$\begin{aligned} c_0 &= \frac{4}{7} \\ c_{90} = c_{45} &= \frac{1}{7} \end{aligned} \tag{22}$$

Thus in terms of volume fractions the anisotropy ratio is

$$\frac{c_0}{c_{90}} = 4 \tag{23}$$

From (13) – (17) it follows that the tensile uniaxial and shear strengths are given by

$$\begin{aligned} T_{11} &= \frac{13}{21} T_{tow} \\ T_{22} &= \frac{13}{63} T_{tow} \\ S_{12} &= \frac{4}{21} \sqrt{\frac{T_{tow} C_{tow}}{3}} \end{aligned} \tag{24}$$

With similar forms for C_{11} *and* C_{22}.

From (24) the anisotropy ratio in strengths has the value

$$\frac{T_{11}}{T_{22}} = 3 \tag{25}$$

Thus the degree of strength anisotropy is a little less than the anisotropy ratio in the volume fractions (23) but both are highly anisotropic in this example.

To go further with a general example take realistic tow strengths for IM-7 like carbon fiber polymeric matrix unidirectional strength properties as

$$\begin{aligned} T_{tow} &= 2700\ MPa \\ C_{tow} &= 1700\ MPa \end{aligned} \tag{26}$$

For the orthotropic laminate specified by the volume fractions (22) the condition is effectively that of 4/7 of the lamina being in a quasi-isotropic arrangement with the additional 3/7 of the lamina added to the 0 direction lamina. The uniaxial and shear strengths are then given by

$$\begin{aligned} T_{11} &= 1671\ MPa \\ C_{11} &= 1052 \\ T_{22} &= 557 \\ C_{22} &= 351 \\ S_{12} &= 236 \end{aligned} \tag{27}$$

The corresponding quasi-isotropic problem properties are given by

$$\begin{aligned} T &= 900\ MPa \\ C &= 567 \\ S &= 412 \end{aligned} \tag{28}$$

Finally the quasi-isotropic properties (28) are put into the failure criterion (1) and the orthotropic properties (27) are put into the orthotropic failure criterion (3) giving the resulting failure envelopes shown in Fig. 2, with σ_{12} taken as zero.

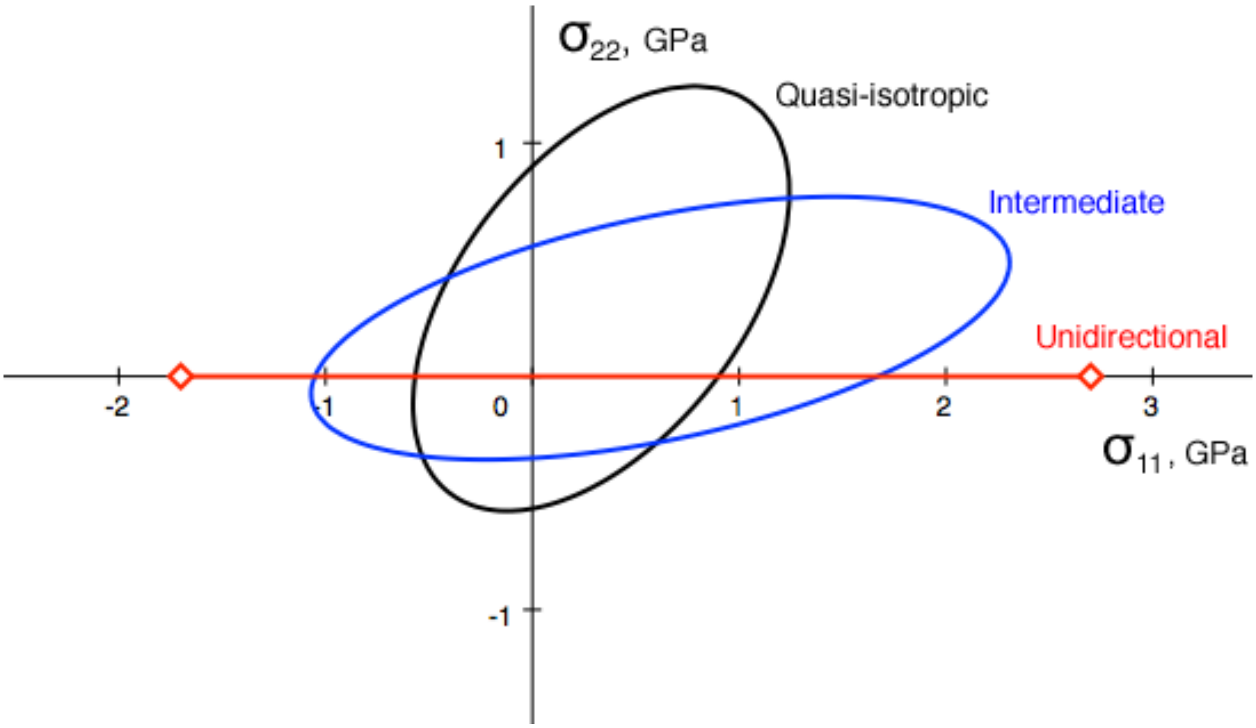

Fig. 2 Failure for fiber dominated unidirectional, intermediate, and quasi-isotropic laminates

Also shown in Fig. 2 is the representation of the unidirectional tow properties given by (26). Fig. 2 graphically and dramatically shows the progression starting from the unidirectional fiber composites strengths going to and through an intermediate form and finally coming to the other limiting case, the quasi-isotropic form. This fiber dominated formalism would not be expected to be realistic near the unidirectional limiting case since it does not include the transverse matrix controlled effects. Other than the need for the micro scale matrix controlled properties the present failure theory applies to and for all orthotropic layup patterns.

Complete and comprehensive design applications would involve trade offs between specification of the volume fractions for the four fiber orientations using the lamination rules (13) – (17) and then the failure criterion (3). Unsatisfactory results would then require further iteration. Using all this in a numerical program for analysis of an entire structure would complete the process. For this to occur efficiently and effectively it would require a sophisticated numerical algorithm. This new lamination/strength theory provides the vital link in the whole process.

With regard to applications and design for applications, it would not be appropriate to use the present fiber dominated methodology for problems having any of the three lamina volume fractions near zero. That condition would require some explicit account of the matrix properties. Also, in any actual application it would always be necessary to have experimental certification of the general results.

Further Verification of the Orthotropic Failure Criterion

Quasi-isotropic laminates are a special case of orthotropic laminates. The previous verification with test results was given in Ref. [1] for the quasi-isotropic case. That comparison was for a biaxial spread of data, mainly in the range of compressive-compressive stresses for the principal stresses applied to the laminate. The present verification will involve the state of uniaxial tensile stress at failure.

The rationale behind the derivation of the formulas (4) and (5) relating tow failure properties to uniaxial laminate failure properties is as follows. In the tow type failure specimen the strain at failure is given by

$$\varepsilon^f = \frac{T_{tow}}{E_{tow}} \tag{29}$$

where E_{tow} is the elastic modulus of the impregnated tow material. In the quasi-isotropic laminate at failure the strain in the lamina that first fails is taken to be the same strain as occurs in the tow specimen. So then for the quasi-isotropic laminate in uniaxial tensile stress

$$\frac{T}{E} = \varepsilon^f \tag{30}$$

with E being the laminate elastic modulus. Thus from (29) and (30)

$$\frac{T}{E} = \frac{T_{tow}}{E_{tow}} \tag{31}$$

Rearranging (31) gives

$$\frac{T}{T_{tow}} = \frac{E}{E_{tow}} \tag{32}$$

From (32) it follows that if $E/E_{tow}= 1/3$ can be proven from testing data then it further follows that $T/T_{tow}= 1/3$ thus verifying (4). Similar arguments would follow for the compressive case of (5).

It is much easier and more reliable to measure non-destructive elastic properties than it is to measure failure properties. Thus the proof of $E = E_{tow}/3$ for the quasi-isotropic laminate will suffice to prove (4) and (5).

The elastic properties for a quasi-isotropic laminate have been quite exhaustively measured by Van Otterloo and Dayal [4]. The values for the elastic modulus E were measured by both ultrasound and mechanical means in 11 different directions in the layup. The mean value for E from the 22 tests was

$$E = 58.5\ GPa \qquad Measured \tag{33}$$

If only ultrasound data were used the mean was 63.1 GPa.

The corresponding modulus of the tow material was measured as $E_{tow}= 178.4$ GPa. Using the formula

$$\frac{E}{E_{tow}} = \frac{1}{3} \tag{34}$$

which is from Ref. [1], Eq. (31), with $E_{11} = E_{tow}$ for the fiber dominated system then gives

$$E = 59.5\ GPa \qquad Predicted \tag{35}$$

Thus the comparison of (33) and (35) provides the further verification of the failure theory.

The Next Step

The basic research problem of the development of failure theory for fiber composite laminates has now been completed and consummated. This does not however terminate interest in the problem. It actually represents the beginning of a new phase to be used in the applications of fiber composite materials. A major step in that direction will be to develop a complete and comprehensive design methodology for fiber composite laminates. Using the results of this paper, this consolidation will be given in a subsequent paper and it will involve both the stiffness and the strength of fiber composite laminates in a form directly appropriate for applications.

References

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Richard M. Christensen

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