VI. CRITICAL EXPERIMENTAL AND THEORETICAL TESTS FOR FAILURE CRITERIA

The term "critical tests" implies more than just a routine comparison with some convenient data. First, a candidate failure criterion should undergo a rigorous examination of its physical and mathematical basis. Then it should be experimentally evaluated using only the highest quality testing data from carefully selected, homogeneously applied stress states. Such formal and focused probes are necessary if realistic (and reliable) failure forms are to be successfully identified.

All of this is best preceded by a reading of the very long and highly unusual search for failure criteria for isotropic materials. The present account will begin with a brief, telescoped history and consequent status in order to provide the proper background for the evaluation.

The Problem

Perhaps the simplest imaginable failure criterion for all homogeneous and isotropic materials would be that of a critical value of the total stored energy in the material. Even though this notion was dispelled at least one hundred and fifty years ago, it is frequently rediscovered and enthusiastically proposed.

One of several roadblocks for this simple energy idea is that the yielding of very ductile metals depends only upon the critical value of the distortional part of the total energy, not the total energy itself. This of course ties in with the fact that dilatational stress states cannot move dislocations, only shear stresses can do so. However, a distortional energy criterion (Mises criterion) does not work for anything but a very ductile material, as is easily demonstrated. So something far more sophisticated than an energy criterion must be required.

After languishing for more than one hundred years, the original Coulomb criterion was put into a very utilitarian form by Mohr. At the time, the resulting Coulomb-Mohr form was widely thought to have all the answers. That transpired a little less than a hundred years ago. But that too turned out to be a false hope. Maximum normal stress and maximum shear stress criteria also proved to be disappointing.

In the years following the notable Coulomb-Mohr effort and over some very long passage of time, the search for failure criteria seemed to degenerate into essentially just postulating forms. These may have had analytical appeal to the originators but they had little or no physical foundation for support and further development.

In simplest terms, stiffness and strength (and toughness) have independent properties. The underlying basic problem is to express these independencies in full tensor forms and to specify the explicit mechanical properties in each that control three dimensional behavior. Of the following three areas of analytical mechanics characterization: stiffness, strength, and toughness, only that of strength still has unresolved problems, after all this elapsed time. There is a broad lack of understanding with respect to the three dimensional characterization of failure conditions i. e. failure criteria. Consequently this technical area remains in a state of uncertainty and disorganization. The other two areas, stress-strain relations (stiffness) and fracture (toughness), are extremely highly developed.

With the historical results for failure criteria appearing to be so meager and unproductive, there is a legitimate question as to whether the problem may be too difficult to ever yield a general solution. Perhaps it always will be approachable only on an empirical, case by case basis.

There is however a completely contrary and opposing point of view. This view sees the historical efforts as having provided a promising “foothold” on the problem, to be used to advantage when the opportunity should arise. In particular, the Mises criterion, despite its restrictions, could serve very well to anchor a more general approach.

Rather than treating failure criteria as merely an adjunct to the stress strain relations, the two should be recognized as being of completely different character thereby requiring absolutely independent constitutive developments. That is the direction followed here and in supporting works. The technical derivation, showing the physical basis, will be summarized, followed by the evaluations of the resulting failure criteria using specific sets of data.

Emphasis will be given to isotropic materials. This is not just because they are so overwhelmingly important, but also because they give the guidelines for all other cases. However, anisotropic, composite materials will also be given due attention.

Isotropic Materials, Theoretical Assessment

Failure criteria of the type considered here are not needed for all materials. For example, elastomers and "soft" biological tissues probably have no need for explicit three dimensional failure criteria. On the other hand, for engineering materials with essential stiffness characteristics, there are limits to this performance and failure criteria are of vital importance in applications. Mainly those materials that possess a range of linear elastic behavior (or nearly so) are those that require the characterization of the limits of that performance. These include most metals, the glassy (but not rubbery) polymers, some wood products, ceramics, glasses, many geological materials, fiber composite materials and others as well. Many or most of these materials are isotropic or nearly isotropic.

The requisite characteristic is that of an effectively linear elastic behavior up to but not beyond some load level. Failure and linear elasticity share a special type of inverse and complementary relationship. The ideal linear elastic behavior is terminated by the failure and all of the nonlinearity is subsumed into the failure criterion. Historically, linear elasticity theory was the progenitor of all classical field theories, but the related failure theory is required to complete the characterization.

A failure criterion takes the form of a surface in stress space. The stress space can either be the 3-space of the principal stresses, or the larger space of the full stress tensor, or smaller sub-spaces. The comparison with geometry is obvious. A geometric surface in 3-space is described by a scalar function of the coordinates $\phi(x_i)=1$. In the stress context, the problem is to find the scalar equivalent for the failure criterion problem, $\phi(\sigma_{ij})=1$, giving the failure surface in stress space.

So the problem comes down to finding the proper form for the scalar function here called the scalar potential of failure. The failure potential is not to be confused with the plastic potential which was treated in earlier papers and has a different meaning and use.

The form of the failure potential will be taken from the same mathematical representation as that used with the elastic energy potential since it too is a scalar and in some broad sense related. However, significant and formative differences between the two will arise immediately thereafter. As with elastic energy, take ϕ as a polynomial expansion in the invariants of the stress tensor, as appropriate to isotropy. This then is the method of polynomial invariants, giving

$$\phi = a_0 + a_1 I_1 + a_2 I_1^2 + a_3 J_2 + --- \tag{1}$$

I_1 is the first invariant of the stress tensor and J_2 the second invariant of the deviatoric stress tensor, as

$$
\begin{aligned}
I_1 &= \sigma_1 + \sigma_2 + \sigma_3 \\
J_2 &= \frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]
\end{aligned}
\tag{2}
$$

where principal stresses are used. See Section II for the more general case not involving principal stresses. The expansion is terminated at terms of second degree, the same as with the elastic energy representation. Parameter a_0 merely establishes the datum and is not needed here. Sometimes a coefficient different than 1/2 is used in the J_2 expression, which is alright so long as it is used consistently.

In the case of the elastic energy potential, the resulting form must be positive definite and therefore a_1=0, but for the failure potential there is no such requirement and so $a_1 \neq 0$. However, for a physical condition that the homogeneous and isotropic material not fail under hydrostatic compression, it is required that a_2=0, leaving the failure potential as

$$\phi = a_1 I_1 + a_3 J_2 \tag{3}$$

Then the failure criterion becomes

$$a_1 I_1 + a_3 J_2 \le 1 \tag{4}$$

Finally, evaluating a_1 and a_3 in (4) in terms of the uniaxial tensile and compressive strengths, T and C, gives

$$\left(\frac{1}{T} - \frac{1}{C}\right) I_1 + \frac{1}{TC} J_2 \le 1 \tag{5a}$$

or

$$\begin{aligned} &\left(\frac{1}{T} - \frac{1}{C}\right)(\sigma_1 + \sigma_2 + \sigma_3) + \\ &\frac{1}{2TC}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \le 1 \end{aligned} \tag{5b}$$

As the overall failure criterion, (5) is necessary but not sufficient. Sufficiency requires that a competitive fracture criterion in the more brittle range of behavior also be satisfied. From Section II this is given by

$$\left.\begin{array}{l}\sigma_1 \leq T \\ \sigma_2 \leq T \\ \sigma_3 \leq T\end{array}\right\} \quad if \quad \frac{T}{C} < \frac{1}{2} \tag{6}$$

There is both geometric and physical significance to the commencement of the fracture criterion at T/C=1/2. It is at this value of T/C that the three fracture planes (6) in principal stress space are just tangent to the paraboloid (5) of the failure potential. The fracture criteria (6) then introduce the occurrence of "corners" into the yield/failure surface characterization for T/C<1/2.

The forms (5) and (6) comprise the complete, two property failure criterion for homogeneous and isotropic materials. In effect, the failure criterion and the elastic energy (from which the stress-strain relations follow) are joint constitutive relations. They are independent, but one without the other is incomplete. The elastic energy has two properties (E and ν) and one condition (positive definiteness) while the failure potential also has two properties (T and C) and one condition (no failure under hydrostatic compression).

Derivatives of the failure potential with respect to stress can be used to gain information about the orientation of the failure surface. However that topic is beyond the scope of what is being collected here.

Putting the above results aside until experimental evaluation in the next sub-section, now a contemporary example of a postulated failure criterion will be given as a case study in contrasts. A major aerospace research group has for several years advocated that the material failure criterion (for the isotropic matrix phase in composites) be written directly in uncoupled, two parameter form as

$$\begin{aligned} I_1 &\le \alpha \\ J_2 &\le \beta \end{aligned} \tag{7}$$

The first of these is said to associate with a fracture behavior and the second with yielding and flow. Taking both α and β to be calibrated by both T and C then gives

$$\begin{aligned} I_1 &\le T \\ J_2 &\le C^2 \end{aligned} \tag{8}$$

It is not clear what the physical basis of (7) and (8) may be. The equations merely reflect that J_2 applies to ductile metals and there also are claims (erroneous) that I_1 completely covers brittle materials, so the proponents take both of them as independently and simultaneously required for all glassy polymers (and by extension for all materials). This is typical of the many schemes that have been tried. Certainly all obvious ways of combining the invariants (including I_3 or J_3) have been proposed and in some cases claimed to be the long sought form for failure criteria. Most of these attempts do not receive a critical comparison with data on well characterized homogeneous and isotropic materials. Rather, they usually involve comparisons with a window of data from non-uniform stress states in complicated forms or structures where many other factors are also in effect, but not acknowledged or even recognized.

With regard to the possibility of forms (8) being a useful, well posed failure criterion, consider the following. One of the basic tenets of materials science is that superimposed pressure increases the strength of the material. This happens in all cases except the limiting case of a Mises material. The failure criteria derived here, (5) and (6), embody this effect in an interactive manner. In contrast, the forms (8) predict that pressure has no effect at all on the uniaxial compressive strength nor on the shear strength, while they give an unusual prediction for the effect of pressure on the uniaxial tensile strength.

Even more objectionable is the prediction of (8) in the case of the ductile limit at T=C. For two dimensional biaxial stresses, (8) gives only a portion of the usual Mises ellipse because the first of (8) truncates it by a straight line cutoff in the first quadrant. In the range of polymer behavior, (8) still exhibits the same characteristic. At T=C the derived failure criteria (5) and (6) directly recover the full Mises criterion.

All of these difficulties with forms (7) and (8) still occur when the invariants are interpreted in terms of strains rather than stresses. This is easily shown by substituting the isotropic stress-strain relations into (7), whichever way they are specified, to directly obtain the other form. Thus (7), whether in their stress or strain forms, fail to pass standard tests of physical consistency. All tentative failure criteria should be subjected to conceptual tests of the types just discussed. The cloud of ill-conceived failure criteria over the years has done much to obscure and impede progress.

There also have been a few very clever constructs, such as the Coulomb-Mohr form, but unfortunately they do not coordinate at all well with the physics of failure. This will be further shown through comparisons with data in the next sub-section since the C-M criterion is by far the most substantial and prominent of all historical forms.

Experimental Evaluation (Isotropic)

Return now to the isotropic material failure criteria (5) and (6). As a conjecture, the form (5) has been known for a very long time, at least from about the time of Mises. The derivation of it given here adds substance and context, revealing its dual relationship to energy. It is however the coordination of (5) and (6) that provides the comprehensive core extending from ductile limit to brittle limit. Nevertheless, none of this is of any consequence unless it reflects physical reality. The criteria (5) and (6) must now pass tests of evaluation with meaningful data or be rejected.

If the objective were merely to characterize failure for a single material type, epoxy polymers for example, then that could not really be considered as developing failure criteria. In reality it would just be "curve fitting" for that material with all the hazards and pitfalls for anything but the simplest interpolations. To validate a failure criterion it is necessary to test it against widely different classes of materials. Otherwise there would be no confidence in its general applicability. The qualifier "general applicability" refers not only to different types of materials, but also to all possible stress states for each.

More specifically, for isotropic materials it is necessary to cut across the full range from very ductile to very brittle materials. The extremes are quite apparent, a very ductile metal should be used to test the one extreme and most likely a very brittle geological material should be tested for the other extreme.

The intermediate case is somewhat more problematic. However, at least two possibilities are at hand for a material that is neither very ductile nor very brittle. These two would be cast iron and very glassy, un-toughened polymers. Of the two, there appears to be more high quality data for cast iron and it is what will be included here. Although cast iron is loosely thought of as being brittle, that is only so in comparison with ductile steels. Over the full range of ductile/brittle behavior, cast iron lies as ideally intermediate, for this purpose. This will be explained more fully later.

Before looking at individual data cases one more testing requirement should be observed. It is very important that testing results not come from conditions containing steep stress gradients. Only homogeneous or nearly homogeneous stress states provide the clarity level consistent with seeking strength type mechanical properties for homogeneous materials.

The initial test case concerns the ductile limit at T=C. There are several different metals that could be used to generate data. The first substantial and enduring testing results were those of Taylor and Quinney [1] in 1931. They used the tension-torsion of thin walled tubes to generate plots of normal stress σ_{11} versus shear stress σ_{12}. The two theoretical forms of interest are the Mises criterion and the Tresca criterion. The Mises criterion is the form from the general failure criteria (5) and (6) at T=C and it also represents a distortional energy criterion. The Tresca criterion is that of the Coloumb-Mohr theory at T=C, and it also represents a maximum shear stress criterion.

The comparison between the two theories and the ductile metals data are as shown in Fig. 1.

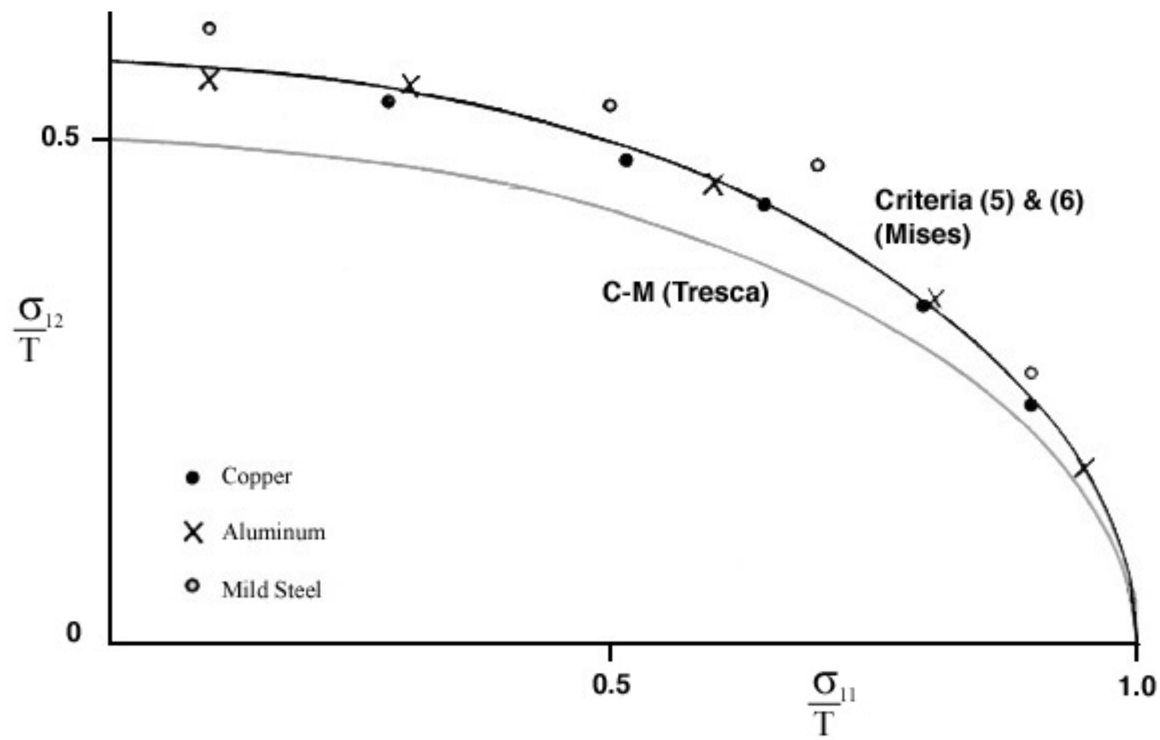

Fig. 1 Taylor, Quinney [1] failure data for ductle metals, T/C=1

The data clearly favor the Mises criterion and thereby the general criteria (5) and (6). G. I. Taylor was one of the great scientists of the twentieth century. These results from him have been substantiated many times and are now considered to be classical. R. Hill [2], who also was and is a world class contributor, affirms this status for these results.

Next consider the intermediate case which is in between extreme ductility and extreme brittleness. This case is that of a particular type of cast iron. In such cases the T/C ratio is about 1/2 to 1/2.5 which is effectively in the middle range between T/C=1 for ductile metals and T/C of about 1/15 or even much less for brittle geological materials. The biaxial cast iron failure data in Fig. 2 are by Cornet and Grassi [3], the first of whom was a careful experimentalist at U. C. Berkeley. The T/C value is 1/2.16.

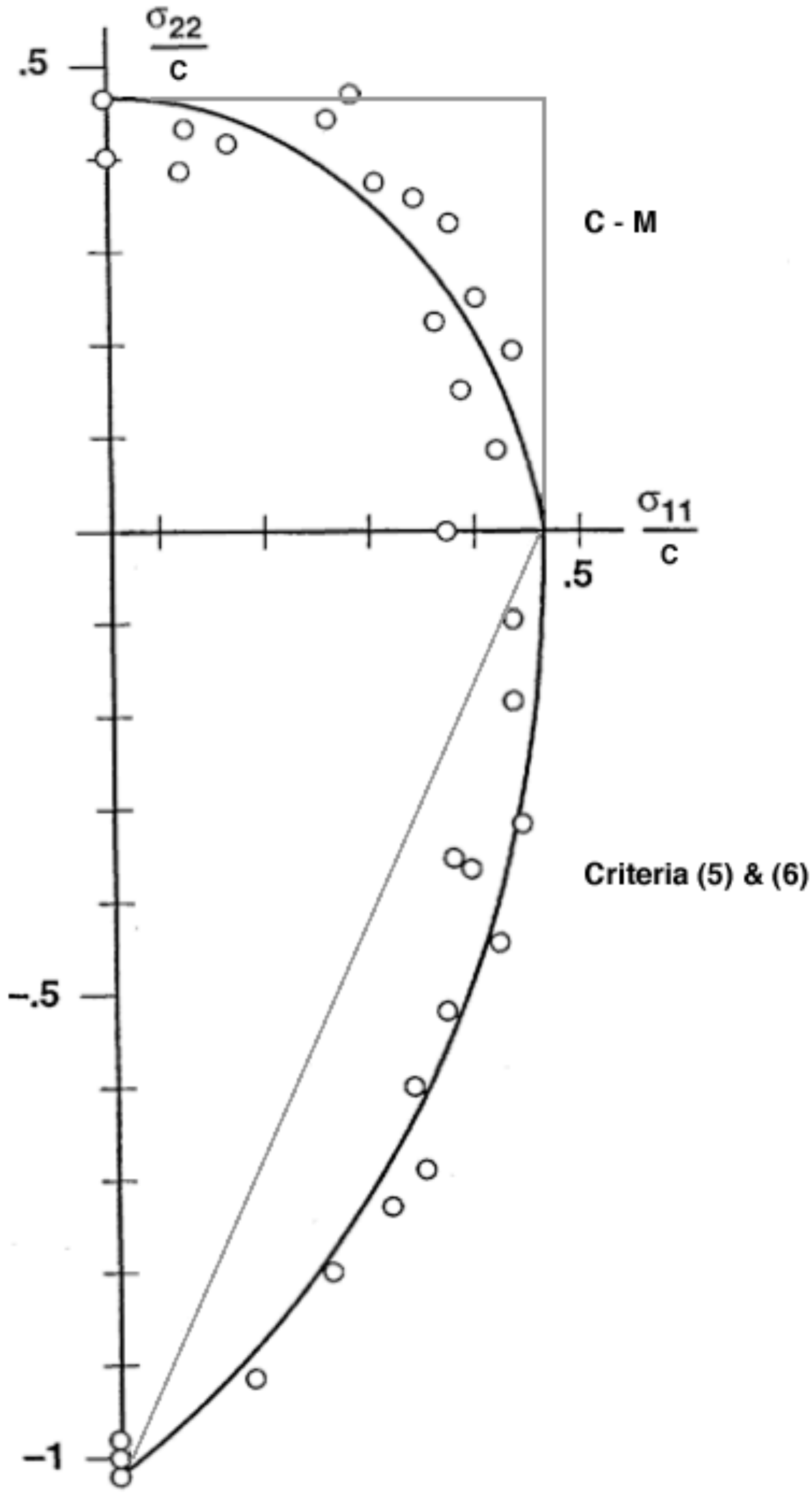

Fig. 2 Biaxial failure data on iron, Cornet and Grassi [3], T/C=1/2.2

The comparison of the data with failure criteria (5) and (6) is favorable while the comparison with Coulomb-Mohr is unfavorable. These data have effectively been reproduced by several other investigators for other types of cast iron. The fracture criterion (6) is just barely discernable as a slightly flattened "spot" on the failure envelope of Fig. 2. The Mises or Tresca criteria by themselves would be completely unsatisfactory in this middle range case.

As the third critical test case, a very brittle material must be used. Reliable failure data was generated on Dolomite by a respected geophysicist at M.I.T. These results are from Brace [4] on samples considered to be close to isotropic, and with T/C=1/14.9.

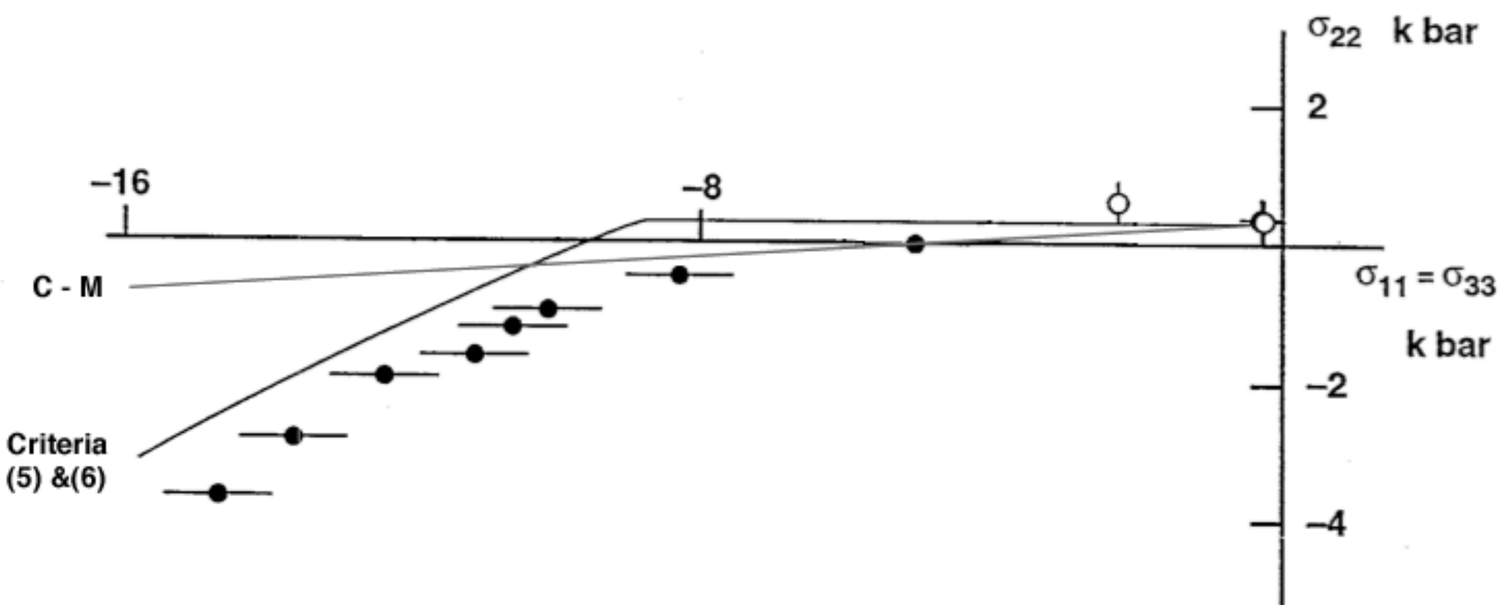

Fig. 3 Tri-axial failure data for Blair Dolomite, Brace [4], T/C=1/15

The failure criteria (5) and (6) provides a good correlation with the data in Fig. 3, whereas Coulomb-Mohr fails to follow the downturn in the data and merely projects a straight line failure envelope.

The upper envelope shown in Fig. 3 is that due to the fracture criterion (6). The experimental uncertainty along the horizontal axis is quite considerable and within the proximity of the predicted failure envelope. Probably the most difficult (and uncertain) testing result is that for uniaxial compressive strength C. If the value of C given in the reference were reduced by 10% the prediction from (5) for the downturn region would fall within the data scatter.

If the stress states were taken with σ_{11} and $\sigma_{22} = \sigma_{33}$ the theoretical envelope would have the down turn just inside the data, rather than just outside the data, as in Fig. 3. If one extends the axes in Fig. 3 one sees the tremendous divergence between the Coulomb-Mohr theory on the one hand and the data and the present theory on the other hand. The downturn in the data and theory shown in Fig. 3 is not surprising. The same effect shows up in The Brittle Limit. It should be noted that the axes labels in Fig. 3 are different from those used by Brace in order to follow common convention.

A comprehensive failure theory must span failure mode types from ductile flow to completely brittle disintegration. It is nearly inconceivable that this could be accomplished without a formal and rigorous framework for treating ductile versus brittle behaviors. The present failure theory, which culminates in the explicit criteria (5) and (6), does have an underlying ductile/brittle formalism. This is mentioned in Section II and presented in the papers upon which Section II is based. The complete formalism gives not only the failure envelopes, but also the division of domains into ductile versus brittle regions. This will be briefly recalled here since it too is amenable to experimental verification or repudiation.

The concept of the ductile/brittle transition is basic to materials science. Usually this is posed as a temperature effect and the ductile/brittle transition temperature can be located by scanning failure type against temperature change. The same approach can be taken with a pre-imposed state of pressure. Sufficient pressure can convert a perceived brittle material into a effectively ductile material and there is a transition from one state to the other.

This interpretation of the ductile/brittle transition admits extension to a variety of other conditions. For example, any particular state of three dimensional stress dictates whether the particular failure mode is ductile or brittle. For a given material, predominantly tensile stress states are far more likely to be brittle in failure than are predominantly compressive stress states. Whether a particular failure mode will be ductile or brittle depends explicitly and crucially upon the stress state itself, as well as the material. Conversely, for a particular stress state, one can scan across various materials types to determine the ductile versus brittle nature of the failure mode.

Following the above prescription, for any particular stress state the material type will be varied by varying the T/C ratio, and then the failure mode type, ductile or brittle, will be identified. From a rather lengthy derivation in the papers in Section II the ductile/brittle transition follows from the intersection of failure criteria (5) and (6) as

$$\begin{aligned} I_1 &= 3T - C \qquad\qquad D/B\ \mathit{Transition} \\ &= (T+C) + 2(T-C) \end{aligned} \tag{9}$$

where $I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}$ is the first invariant for the stress state of interest that is on the failure envelope from (5). The ductile and brittle conditions are then given by

$$
\begin{aligned}
I_1 &< 3T - C \qquad \textit{Ductile} \\
I_1 &> 3T - C \qquad \textit{Brittle}
\end{aligned}
\tag{10}
$$

where again I_1 is from (5). Relation (9) is physically meaningful only if the solution for T/C falls in the range from 0 to 1. Otherwise (10) shows the condition to always be ductile or brittle for all allowable values of T/C.

It would be expected that the criterion for the ductile/brittle transition would involve the first invariant part of the total stress state at failure. This can be thought of as algebraic pressure and that has an intimate relationship with temperature in changing the state of substances.

A simple example will illustrate the use of (9) and (10). Take the state of uniaxial tension. For this case the solution from (5) is almost trivial, it is $I_1=T$, and substituting that into (9) gives

$$
\frac{T}{C} = \frac{1}{2} \qquad \textit{Uniaxial Tensile D/B Transition}
$$

For materials with T/C>1/2 the tensile failure is predicted by (10) to be ductile and for T/C<1/2 the tensile failure is predicted as brittle. This is in accordance with most experimental observations for a wide variety of different materials types. Most metals and most polymers are found to be on the ductile side of the predicted D/B transition at T/C=1/2, while a few metals, a few polymers, and all ceramics, glasses, and geological materials are on the brittle side. The three preceding data test cases are in agreement with this classification of behavior. The complete spectrum of ductile/brittle uniaxial tensile behaviors is yet another aspect of experimental corroboration of the general theory.

For uniaxial compression, from (5) $I_1 = -C$ and that into (9) gives T/C=0. Thus uniaxial compression is always ductile, but becomes borderline brittle as T/C approaches the brittle limit.

The range covered by this failure theory is $0 \leq T/C \leq 1$. An interesting sequence of quite simple stress states that covers the full range of ductile/brittle T/C's is given in the table below.

Stress State	**D/B Transition** T/C=	**D/B Behavior** T/C
Equal Biaxial Tension	1	<1 Brittle
Uniaxial Tension	1/2	>1/2 Ductile <1/2 Brittle
Simple Shear	1/3	>1/3 Ductile <1/3 Brittle
Uniaxial Compression	0	>0 Ductile

It follows from (10) that equal triaxial tension is brittle for all values of T/C and equal biaxial compression is ductile for all values of T/C. It also is recalled that there is no failure under equal triaxial compression.

The two uniaxial stress states have already been discussed. The state of simple shear is also of importance. For simple shear $I_1=0$ and then (9) gives the D/B transition as being at T/C=1/3. From the failure criteria (5) and (6) at T/C=1/3 it is found that the shear failure stress is exactly the same as that of uniaxial tension at this value of T/C. Shear stress rotated 45 degrees is equivalent to a tensile stress component and an orthogonal compressive stress component of the same magnitude. This shows that the compressive stress component has no effect on the failure mode, it is entirely caused by the tensile stress component. This is symptomatic of brittle behavior.

Perhaps the most interesting case in the above table is that of the equal biaxial tension. The solution from (5) for $\sigma_1 = \sigma_2$ and $\sigma_3 = 0$ gives the tensile root as

$$I_1 = 2\left(T - C + \sqrt{T^2 - TC + C^2}\right)$$

and this into (9) gives the D/B transition as being at T/C=1. Now a material such as aluminum or steel with T/C=1 is certainly thought of as being a ductile material, and it most assuredly is ductile in uniaxial tension. But the result in the Table shows that such normally and nominally ductile materials but in a state of equal biaxial tension are actually at the transition between being ductile and brittle. Experimental pressurization of ductile steel spherical pressure vessels reveals failure modes that are more suggestive of brittle behavior than of ductile behavior. Typical failure modes for this situation are as shown below, as given by Talja et al [5].

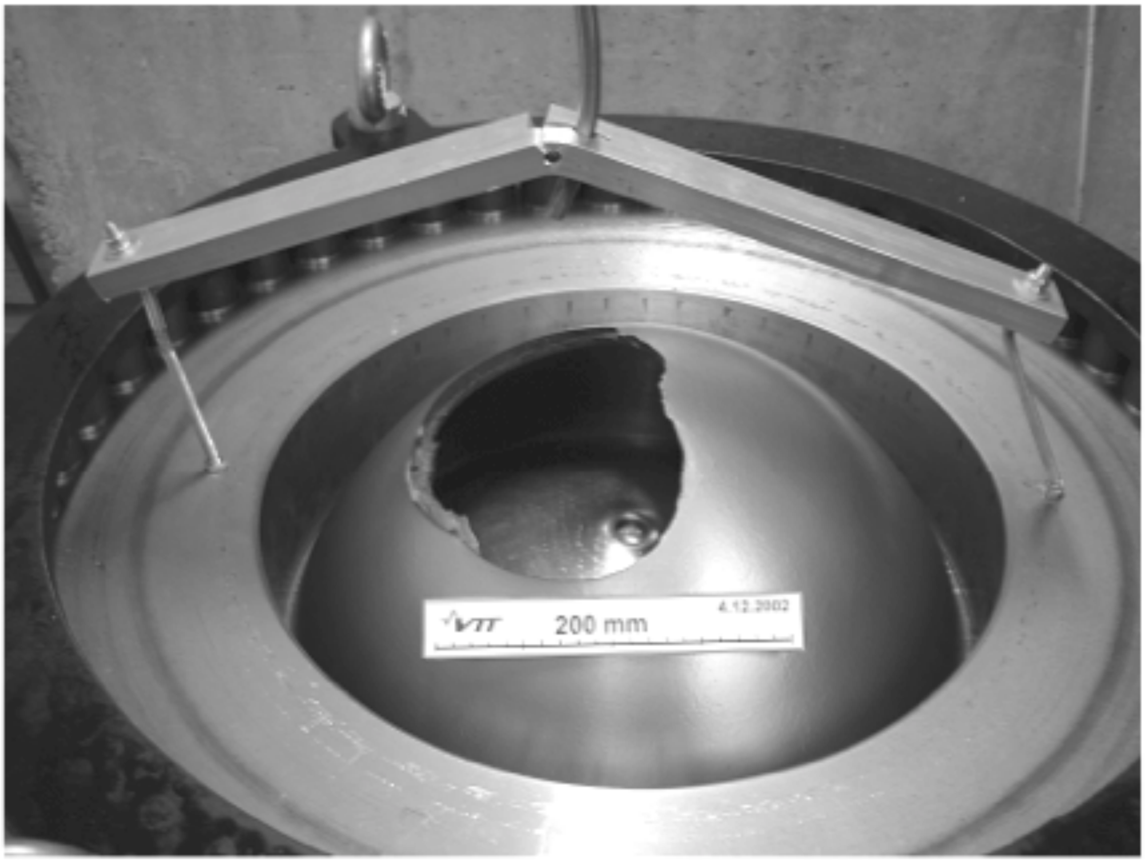

Fig. 4 Spherical steel pressure vessel failure mode, Talja et al [5]

In principal stress space, the failure criterion (5) takes the form of a paraboloid, while the fracture criteria (6) are planes that cut off portions of the paraboloid. The ductile/brittle transition specification (9) is that of a plane normal to the axis of symmetry of the paraboloid. It divides the paraboloid into ductile and brittle regions. For T/C<1/2 it also cuts across the three fracture planes thus giving failure modes of brittle fracture and of ductile fracture types.

Isotropy Conclusion

There are many other isotropic material failure criteria that have been proposed and (inappropriately) used. These can easily be evaluated using the data cases just discussed. For the Mises and Tresca criteria, only the Mises form is completely satisfactory for ductile metals, but both are in serious error for everything else. Of the two parameter variety, these include the maximum stress (tensile and compressive), the maximum strain, and the Drucker-Prager forms. All of these compare very poorly with the data cases. Equally importantly, all of the above criteria fare no better in assessing their theoretical roots. The inadequacy of the Coulomb-Mohr form has already been shown here many times.

Other data evaluations, for other materials types including polymers and ceramics are given in the papers cited in Section II. Those results are compatible with the conclusions reached in this section.

Three parameter forms have not been considered here because of the success of the two property theory that has been the main focus. In recent times three parameter forms have mainly been used in problems of crack formation in ductile metals after experiencing extreme plastic deformation. Typically the third parameter involves the third invariant I_3, often in the form of the Lode angle.

The failure theory developed here is for a totally different purpose than that just mentioned. The present work is relevant to the onset of dominating irreversible deformation, whether it be due to the inception of major plastic flow, as a form of failure, or explicit, brittle failure itself, or due to any other mechanism.

The overall conclusion from this failure theory and its experimental evaluation is that the two properties T and C in combination provide a fundamental and comprehensive calibration of materials strength performance. The defining two part failure criterion, (5) and (6), embedding these two properties completes the constitutive specification for homogeneous and isotropic materials.

Fiber Composite Laminates

Fiber composite materials present a particular challenge in characterizing failure because of the complicated microstructures and macro-structures. The fibers are not perfectly straight and continuous with perfectly ordered spacing between them. Another severe complication is that of the extreme anisotropy at all scales. Nevertheless these materials are of importance and require treatment here. The scales of particular relevance are (i) the atomic scale inside the fibers, (ii) the fiber scale itself, (iii) the lamina scale into which the fibers are (nominally) aligned within a matrix phase, and (iv) the laminate scale comprised of many lamina arranged at various orientation angles.

Interest here is with carbon fiber science and technology. At the atomic scale the carbon-carbon bond is one of truly superior capability. The ideal carbon fiber would actually be that of a continuous multi-walled nano-tube. As manufactured, carbon fibers have nano-scale structures that are far different and vastly more irregular than those of nano-tubes. As the result, the fiber scale properties are much diminished from the smaller nano-scale, ideal properties. Correspondingly the strength properties going from the fiber scale to the lamina scale are reduced. So to in going from the lamina to laminates scale, the strength properties are reduced. The pattern is obvious, enlargements in scale can do nothing but cause overall reductions in the strength properties. Since all applications are at the laminate scale, the attention here must primarily be at that scale, although all scales are of great general interest.

Two general approaches are considered here for developing failure criteria and predicting failure at the laminate scale. These two methods are those of progressive damage initiating at the lamina scale and polynomial invariants which is completely at the laminate scale. Both of these methods are described in Section V and will be summarized here. Of course there are many other approaches available. For example there are many different approaches just at the micromechanics level where the individual fibers and the matrix phases are given distinct and different failure criteria. One such criterion was discussed earlier under the isotropic materials heading as an example of an unsatisfactory approach to failure criteria for isotropic materials (intended for the matrix phase in composites).

In progressive damage the lamina scale failure characterization is used to predict laminate failure. The occurrences of essentially matrix controlled failure modes and fiber controlled failure modes in the various lamina must be accounted for sequentially as the load increases until so much damage has built up in the many lamina that the laminate can no longer sustain the load. What sounds simple in concept is not so simple in practice. Specific difficulties and complications with this approach will be described a little later.

In the other approach, polynomial invariants, the failure criterion is completely developed at the laminate scale. The geometric arrangements of the lamina can be used to specify the symmetry properties of the overall laminate. This then is used to develop laminate failure criteria with unspecified properties that must come from calibration with minimal failure data from specific and simple stress states.

In the case of progressive damage for application to carbon fiber composites, there is a useful idealization called fiber dominated progressive damage. In these cases the matrix controlled strength properties are considered to be small enough to allow neglecting them compared with the fiber controlled contributions to strength. This simplifies the procedure but still preserves the basic forms of the failure envelopes in stress space, for fiber dominated systems.

A particularly important layup is that of the quasi-isotropic form involving fibers in equal proportions in the 0, +45, -45, and 90 degree directions. Under a biaxial stress state, σ_{11} versus σ_{22}, fiber dominated progressive damage gives a diamond shaped failure envelope. Unfortunately there are conflicting experimental data sets for this problem. One set partially supports the diamond shaped failure envelope while the other set decidedly does not support the diamond shaped form. This circumstance and these results are described in detail in Section V. So the situation is unsettled. It would be helpful to have some independent evaluation of the method of progressive damage, especially since it is so widely used. Along these lines there is only a small but fairly decisive indicator of the state of effectiveness for progressive damage, which will now be described.

There is a simpler problem with which the method of progressive damage can be conceptually tested. This problem is also for the quasi-isotropic layup, but with a much simpler imposed stress state, namely that of uniaxial tension. Specifically, fiber dominated progressive damage can be used to predict the uniaxial tensile strengths in two cases:

(i) Uniaxial stress aligned with a fiber direction

(ii) Uniaxial stress aligned mid-way between fiber directions

Another way to say this is for (i) the stress is in the 0 degree direction and for (ii) the stress is in the 22.5 degree direction.

In Section V it is shown that fiber dominated progressive damage is equivalent to a laminate criterion of critical normal strain in fiber directions. Applying this criterion to problems (i) and (ii) it is found that for the strength normalized to the 0 degree case there results

(i) $\sigma^T_{11}=1$, 0° direction

(ii) $\sigma^T_{11}= 3/(1+\sqrt{2}) =1.243$, 22.5° direction

Thus the uniaxial strength is predicted to be 24.3% greater in the mid-way between fiber directions than in the fiber directions for the 0/+45/-45/90 degree quasi-isotropic layup.

The same problems can be posed for the 0/+60/-60 degree quasi-isotropic layup. It is found that the mid-way between fiber directions strength is predicted to be 50% greater than in the fiber directions.

These predictions from fiber dominated progressive damage certainly seem to be very un-physical and extremely unlikely, and they cast considerable doubt upon the method of progressive damage. For this reason the method of progressive damage will not be pursued further here. It must be added however that definitive and determining experimental data are not yet available. Until that gap in understanding is closed with unambiguous data a final pronouncement cannot be made

There also is another avenue that could be considered. In the fiber dominated progressive damage method the fiber controlled failure is solely determined by the stress component in the fiber direction with no interaction from the other stress components. This could be modified to bring in the other stress components and thereby change the failure prediction. But it is difficult to see how this could be done in a rational, physical properties oriented manner. More likely, it could only be done as a parameter variation exercise, which would not advance the level of understanding.

Another complication is with the difficulty in testing fiber composites for strength. There should be independent corroboration of critical data sets. Although this is reasonably well satisfied for the various classes of isotropic materials, it is not usually available for evaluating failure criteria for anisotropic fiber composite materials. The latter are extremely difficult to test, at least partially because of the extreme anisotropy.

The negative situation just described with progressive damage leaves only the polynomial invariants method as available for further investigation here. From Section V the in-plane failure criterion for a quasi-isotropic laminate is given by

$$\left(\frac{1}{T}-\frac{1}{C}\right)(\sigma_{11}+\sigma_{22})+\frac{1}{TC}(\sigma_{11}+\sigma_{22})^2+\frac{1}{S^2}\left(\sigma_{12}^2-\sigma_{11}\sigma_{22}\right)\le 1 \tag{11}$$

where T and C are the in-plane uniaxial strengths and S is the in-plane shear strength. Thus there are three properties to be determined. A reasonable estimate from Section V of the shear strength property is given by

$$S^2 = \frac{3}{8}TC \qquad (12)$$

Using (11) and (12) the theoretical form is compared with the biaxial testing data of Welsh, Mayes, and Biskner [6] as shown in Fig. 5.

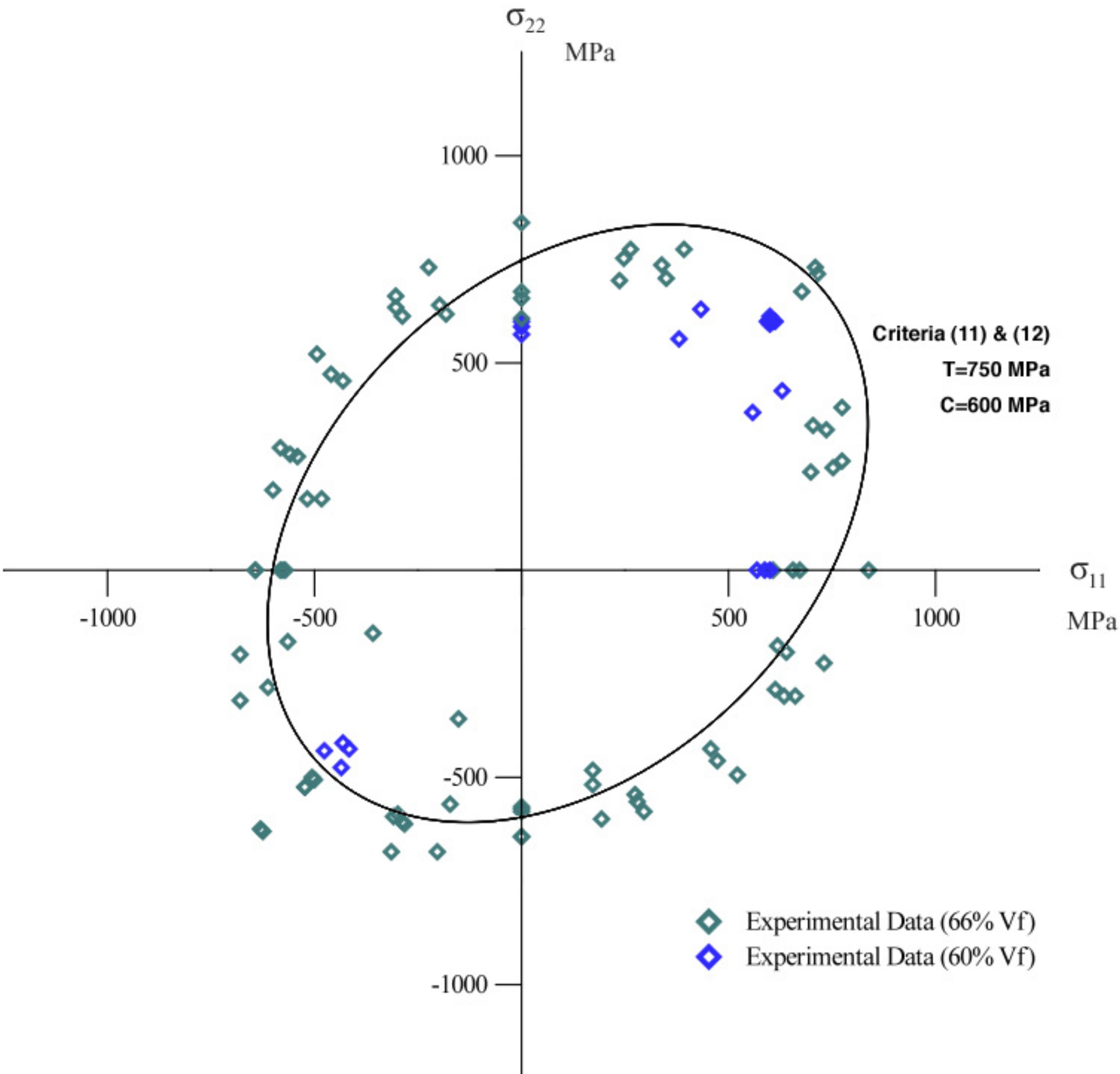

Fig. 5 Quasi-Isotropic laminate failure, data from Welsh, Mayes, and Biskner [6]

The comparison of the polynomial invariants failure envelope with the data is quite good. It is certainly far better than that given by the diamond shaped failure envelope from progressive damage. However, this result cannot be considered as definitive for the reasons already mentioned. Far more data cases are needed before anything definite can be said. Up to date evaluations of the types given earlier by Hinton, Kaddour and Sodden [7] are needed.

The method of polynomial invariants also gives failure criteria for the more general cases of orthotropic laminates, and for delamination as well. These are given in Section V, but as already stated, the major missing piece is that of reliable testing data.

It is apparent and obvious that the development of failure criteria for fiber composite materials is only at a beginning stage compared with that for isotropic materials. In the isotropic materials case it was argued earlier that any proposed failure form could not be considered to be a verified and qualified failure criterion until and unless it is convincingly shown to be applicable to a wide variety of different materials types. Nothing even approaching that situation has been given (here or elsewhere) for the composites case. The only class of composites considered here is that of highly anisotropic lamina arranged in laminate forms, through and including orthotropic symmetry. There is however a considerable advantage for the approach taken here for this particular problem. The method of polynomial invariants for fiber composite laminates did not just coalesce in total isolation from anything else. It was and is a direct generalization of the method of polynomial invariants as successfully applied to isotropic materials.

Much further work in the composites area is needed and will be useful, but there already is a treasury of valuable information from investigations into individual failure modes. Condensing all that information into concise, compact, and especially verified forms for failure criteria is still a missing building block for the general applications of fiber composite materials.

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