XXV. THE DUCTILITY NUMBER ND PROVIDES A RIGOROUS MEASURE FOR THE DUCTILITY OF MATERIALS FAILURE

1. Introduction

There was just one decisive, really pivotal occurrence throughout the entire history of trying to develop general failure criteria for homogenous and isotropic materials. The setting was this: Coulomb [1] had laid the original groundwork for failure and then a great many years later Mohr [2] came along and put it into an easily usable form. Thus was born the Mohr-Coulomb theory of failure. There was broad and general enthusiasm when Mohr completed his formulation. Many people thought that the ultimate, general theory of failure had finally arrived. There was however a complication.

Theodore von Karman was a young man at the time of Mohr's developments. He did the critical experimental testing of the esteemed new theory and he found it to be inadequate and inconsistent [3]. Von Karman's work was of such high quality that his conclusion was taken as final and never successfully contested. He changed an entire course of technical and scientific development. The Mohr-Coulomb failure theory subsided and sank while von Karman's professional career rose and flourished.

Following that, all the attempts at a general materials failure theory remained completely unsatisfactory and unsuccessful (with the singular exception of fracture mechanics). Finally, in recent years a new theory of materials failure has been developed, one that may repair and replace the shortcomings that von Karman uncovered. The present work pursues one special and very important aspect of this new theory, the ductile/brittle failure behavior.

The failure theory of Christensen [4] will be briefly outlined in the next section, Section 2. Then in Section 3 the many evaluations and verifications of this new failure theory will be fully documented, as is absolutely essential for long term credibility. Next, a new method of quantifying the ductility level of the failure will be developed and demonstrated in Section 4. This new measure or index of ductility is named the ductility number, Nd. It is a natural outgrowth from the present failure theory. The following two sections, 5 and 6, contain interpretations and applications of the ductility number, Nd. Section 7 developes important, non-trivial conclusions.

2. Failure Theory

The isotropic materials failure theory was developed and displayed in the recent book by Christensen [4]. It is for materials that after experiencing a range of linear elastic deformation reach their limit of load capacity and fail. The main failure criterion is given by

$$\left(1-\frac{T}{C}\right)\hat{\sigma}_{ii}+\frac{3}{2}\hat{s}_{ij}\hat{s}_{ij}\leq\frac{T}{C} \qquad for\ \ 0\leq\frac{T}{C}\leq 1 \tag{1}$$

where T and C are the calibrating uniaxial strengths, s_{ij} is the deviatoric stress and all forms of stress are nondimensionalized by C through

$$\hat{\sigma}_{ij}=\frac{\sigma_{ij}}{C} \tag{2}$$

There also is an auxiliary and competitive fracture criterion given by

$$\begin{aligned}\hat{\sigma}_1&\leq\frac{T}{C}\\ \hat{\sigma}_2&\leq\frac{T}{C} \qquad for\ \ 0\leq\frac{T}{C}\leq\frac{1}{2}\\ \hat{\sigma}_3&\leq\frac{T}{C}\end{aligned} \tag{3}$$

and it is operative only over the partial range shown for T/C, and this is expressed in terms of the three principal stresses.

The expanded and dimensional form for (1) is given by

$$\left(\frac{1}{T}-\frac{1}{C}\right)\sigma_{ii}+\frac{1}{2TC}\left[\left(\sigma_{11}-\sigma_{22}\right)^2+\left(\sigma_{22}-\sigma_{33}\right)^2+\left(\sigma_{33}-\sigma_{11}\right)^2+6\left(\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2\right)\right]\leq 1 \tag{4}$$

The dimensional forms of the fracture criteria (3) are obvious.

The basic and controlling failure forms are completed by the determining relation for the ductile/brittle transition given by

$$3\frac{T}{C} - \hat{\sigma}_{ii}^{f} = 1 \qquad (5)$$

where the dilatational stresses are those at failure from (1)-(4).

The first term in the ductile/brittle transition (5) represents the materials type. The second term represents the effect of the type of stress state. The latter has the natural and logical form, that of the dilatational stress effect at failure. This form of (5) could not have anything but the dilatational form in the failure criterion (1) or the distortional form involving the deviatoric stresses. The distortional form in (1) has no specific evidence supporting its use, but the dilatational term certainly does.

It is well know that applied pressure has a strong effect on the ductility level of failure. This has been experimentally observed for a very long time. The dilatational term shown in (5) formalizes this effect and provides an intuitive guide on what is actually happening with the ductile/brittle transition. The formal, fully documented derivation of (5) is given in Ref. [4]. It results from the intersection of the consequent different modes of failure that exist through the theory.

These relations (1)-(5) comprise an extraordinarily compact and concentrated set of failure conditions considering the range of materials covered in going from T/C=0 to T/C=1. This is over the range from very brittle materials at the one extreme to very ductile materials at the other extreme. There were many facets of materials failure behavior to be considered in the derivation of these forms. They were fully, even exhaustively considered and accounted for in the two accounts, Refs. [4] and [5] and as surveyed in the perspective overview of Ref. [6].

3. Experimental and Operational Verifications of the Failure Theory

There would be no point in pursuing any further ductile/brittle investigations unless the underlying isotropic failure theory is well grounded and well established. More explicitly, no matter how sophisticated or elegant a failure theory may appear to be, if it isn't verified with hard, clear, high quality experimental data then it isn't worth much of anything, at least not in the engineering applications that are of concern here. This leads to the following standard to be followed: any failure criterion that cannot be proven to be comprehensively correct between and including the limits in $0 \leq T/C \leq 1$ is necessarily empirical and of no use or interest here.

There is only one comprehensive and verified failure theory, that of Ref. [4]. There are surprisingly many confirmations of this failure theory that are scattered around in various papers and in the book [4]. It is important to document their existence and the sources for their accessibility. Except for the first three sources cited below, the other sources are not in any priority order, they are grouped and arranged differently. The references to the specific sources will always be stated and when the book Ref. [4] is cited, the appropriate page number will also be given. The explicit data source references are included in the evaluation references cited below.

- The classical failure data of Taylor and Quinney is for very ductile metals with virtually $T/C=1$ in combined tension and shear stress states. The data strongly corroborates the Mises criterion and not the Tresca criterion, verifying this general failure theory which reduces to the Mises criterion at $T/C=1$. See Book [4], p. 78.

- In the mid-range $T/C=1/2.16=0.463$ an "inoculated" iron, which is much less brittle than standard cast iron, produced corroborating data in the first and fourth quadrants of biaxial stress failure, Book [4], p. 79.

- In the extreme brittle range dolomite has $T/C=1/14.9=0.067$. Fully three-dimensional testing with σ_{22} vs. $\sigma_{11}=\sigma_{33}$ under compressive conditions verifies the general failure criterion prediction, Book [4], p. 79.

The above three cases have data sources that are here considered to be classical because of their importance for understanding materials failure across nearly the entire spectrum of materials types. The respective three data sources are those of Taylor and Quinney [7], Cornet and Grassi [8], and Brace [9].

If there were no other verifications than just these three data cases, they would still make a persuasive argument for the validity of the theory, covering as they do nearly the full range of T/C's. No other failure criterion is even remotely in the same vicinity as this failure theory for these data cases. But it isn't just these three cases, there are more, many more.

- In the early days of rocketry sintered graphite was used as the heat shield in rocket motor nozzles. Graphite in this form has T/C=0.380. It's failure data in the first and fourth quadrants of biaxial stress compared favorably with the theory, J. Engr. Mater. & Tech., 2004, v.126, p.45.

- Polypropylene has T/C=0.818. The uniaxial tensile, compressive and shear strengths under superimposed hydrostatic pressure were successfully predicted by the theory, Book [4], p. 89.

- The theory predicts that the ductile/brittle transition in uniaxial tension occurs at T/C=1/2. The assemblage of world based experience for all the major materials classes is in complete compliance with this division into ductile and brittle groupings, J. Appl. Mech., 2016, v. 83, p. 021001. This is a tremendously strong verification of the failure theory. If one were to pick the single most important fact of the entire verification process, this would have to be it.

- For uniaxial tension and compression for T/C=1 materials the failure angle from the longitudinal direction is predicted to be the octahedral angle, $\varphi=54.7^{o}$. This is in accordance with common experience for very ductile metals, Pro. Royal Society A, 2018, v. 474, p. 20170817.

- For glassy polymers such as untoughened polystyrene at T/C=1/2 the failure angle in uniaxial tension is predicted to be $\varphi=90$, as with common fracture, and this is verified with data, Pro. Royal Society A, 2018, v. 474, p. 20170817.

- Common cast iron is usually noted to have a failure angle of about $\varphi=45^{o}$ in uniaxial compression. The theoretical prediction is $\varphi=41.8^{o}$ at T/C=1/3 and $\varphi=43.0^{o}$ at T/C=2/5, Pro. Royal Society A, 2018, v. 474, p. 20170817.

- Very brittle geological materials undergo a splitting failure mode in uniaxial compression. The failure theory predicts this to occur at

T/C=0. This is at T=0 and C≠0. Furthermore the failure theory predicts that C≠0 because of the stabilizing effect of the mean normal stress (pressure) effect even though the material cannot sustain any tensile stress, Pro. Royal Society A, 2018, v. 474, p. 20170817.

- The theory predicts that the ductile/brittle transition occurs at T/C=1 when in a state of eqi-biaxial tension. This is in accordance with the observed fracture, fragmentation of thin spherical pressure vessels even though the material is nominally very ductile at T/C=1, Book [4],p. 83. See also the website: FailureCriteria.com, Section VI.

- Very ductile metals when supercooled from the molten state form an amorphous material, a metallic glass. One such prominent form, CuZr, has been analysed by molecular dynamics with verification of the failure criterion and the ductile/brittle transition in Christensen, Li and Gao, [10], Pro. Royal Society A, 2018, v.474, p. 20180361.

- The basic formulation of the ductile/brittle transition has been used to derive the ductile/brittle transition temperature for steel, iron, and epoxy. While many factors effect the ductile/brittle transition temperature, this basic, "first order" approach gives values in accord with common experience, J. Appl. Mech., 2016, v. 83, p. 021001.

- If a fiber dominated composite material laminate is arranged in the quasi-isotropic laminate configuration, it's theoretical values for the elastic modulus, Poisson's ratio and the tensile strength comply with the measured values, J. Appl Mech., 2017, v. 84, p. 071007.

- Spallation is due to the occurrence of a pressure wave reflecting from a free surface as a tensile wave. The ductile/brittle behavior theory shows that the reflected tensile wave can cause brittle failure for realistic values of Poisson's ratio even though the material is nominally very ductile at T/C=1, RMC research notes.

- The angle of repose designates the gravity induced equilibrium angle for a mound of any size composed of granular materials. The present materials failure theory taken at T/C=0 shows that the maximum angle of repose is at 45°. This is in accordance with widely observed values, Wikipedia, "Angle of Repose", RMC research notes.

- The failure theory gives the ductile/brittle transition at T/C=1/3 for shear and at T/C=1/2 for uniaxial tension. An epoxy resin and similar resins have about T/C=2/3. The theory then shows that for fiber composites the 3-D transverse tension state produces brittle failure while the shear state remains perfectly ductile. This is compatible with the common transverse cracking in fiber composites while still retaining outstanding longitudinal ductility capability in shear. The latter capability has much to do with the high performance capability of carbon fiber composite materials, Book[4], p. 111 and 112.

- The cohesion and veracity of the developments yet to follow in this paper constitutes another validation of this failure theory.

These examples cover failure behavior for a very wide variety of conditions including the explicit failure behavior and the ductile/brittle characteristics of the ensuing failure. An extremely varied range of materials types have been examined as well as an extremely broad range of stress states. The success under all such conditions amply demonstrates, indeed proves the validity and power of this general failure theory.

4. The Ductility Number, Nd

It would be of major advantage and utility to have a single index that quantifies the state of ductility for each and every stress state that is taken to failure. The concept of ductility as used here must be contrasted with common practice. The usual presumption is that ductility is fully covered and explained by the strain to failure in uniaxial tension. That is an inadequate and misleading concept. It isn't incorrect but it only says something about uniaxial tension and nothing more. What about other states of stress. There needs to be a much broader and more inclusive measure for ductility. The degree of ductility must in some sense comprise a measure of the distance from the ductile/brittle transition in stress space for any state of failure stresses from the failure theory.

Please see the treatment of ductility in Ref. [4] for a fuller understanding of and an operational concept for the physical meaning of the term ductility. Ductility was shown there to be characterized in the region of the deviation from linear elastic behavior for any stress state, not merely and sometimes misleadingly by extravagantly large strains in uniaxial tension.

The key to implementation in this area is the ductile/brittle transition (5) as derived from the general failure theory. In order to develop a general treatment and measure for ductility, rearrange the terms in the ductile/brittle transition criterion (5) to give what will be termed as the ductility number, Nd

$$Nd = 3\frac{T}{C} - \hat{\sigma}_{ii}^{f} - 1 \qquad (6)$$

It follows from the general failure theory, Section 2, that from using (6) there is specified

$$Nd \le 0 \qquad Brittle \qquad (7)$$

and

$$Nd > 0 \qquad Ductile \qquad (8)$$

These two relations provide the basis for proceeding further.

What could be simpler than (7) and (8). If the ductility number is negative the failure is brittle and if positive it is ductile. Using the definition of Nd, (6), it follows that for the three most basic stress states

$$Nd = 3\frac{T}{C} \qquad Uniaxial\ Compression \qquad (9)$$

$$Nd = 3\frac{T}{C} - 1 \qquad Shear \qquad (10)$$

$$Nd = 2\frac{T}{C} - 1 \qquad Uniaxial\ Tension \qquad (11)$$

Therefore the ductility number for each of these stress states remains to depend on the materials type designated by its T/C value.

From uniaxial tension form (11) it follows that Nd=1 at T/C=1. Thus uniaxial tension Nd at T/C=1 is termed as the state of full ductility. It follows that Nd=1 is at full ductility for all stress states just as Nd=0 is at the ductile/brittle transition for all stress states. Furthermore in uniaxial tension (11) gives the ductile/brittle transition at T/C=1/2 in accordance with physical reality.

The complete set of ductile/brittle failure behaviors can now be expressed in the final form

$$Nd < 0 \qquad Brittle \tag{12}$$

$$Nd = 0 \qquad D/B\ \ Transition \tag{13}$$

$$Nd > 0 \qquad Ductile \tag{14}$$

$$Nd \geq 1 \qquad Full\ \ Ductility \tag{15}$$

There is a gradation of ductility levels between Nd=0 and Nd=1 going from no ductility to full ductility.

The ductility numbers designated and controlled by (12)-(15) are illustrated schematically in Fig. 1. As shown, brittleness is by definition the state of no ductility. There probably is no universal form for the Nd variation between 0 and 1 in Fig. 1 but the type shown is likely the common one. It would be straightforward to use the form in Fig. 1 to interpolate the state of ductility from any particular value of the ductility number Nd in the region between 0 and 1. There is no absolute scale for the ductility level. Everything is relative to the state of no ductility (brittle behavior) and to the state of full ductility, as in Fig. 1.

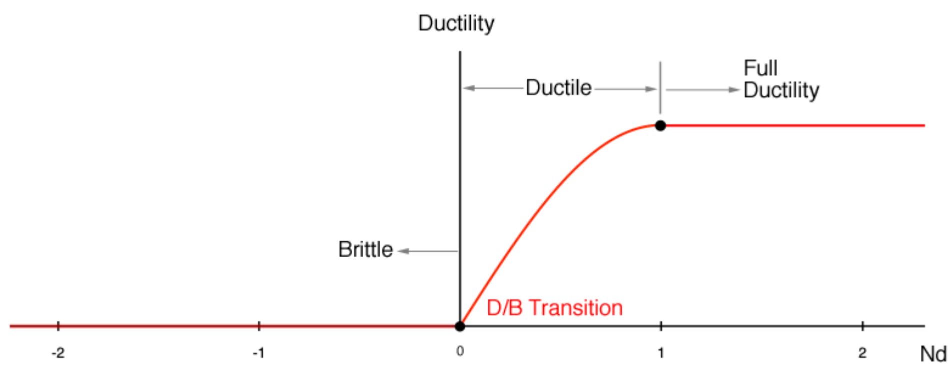

Fig. 1 Ductility levels as determined by the ductility numbers Nd, (6), (12-15)

The hard work was in deriving the governing form (5) for the ductile/brittle transition. The benefit of that effort is the extremely simple and clear characterization in (6) and (12)-(15), and shown in Fig.1.

In Ref. [4] there was introduced a different ductility index called the failure number, Fn. The difference between Nd here and Fn previously is in their different scalings of the same physical effect. They are equally rigorous in their respective significances but Nd is much simpler and far easier to interpret and use. Nd is also much more intuitive since negative values directly designate a state of no ductility and in the other direction Nd represents the nondimensionalized distance from the ductile/brittle transition.

Table 1 shows the spread of ductility numbers for the three basic stress states at five different values of T/C. The values T/C=1/3, 1/2,and 2/3 are special cases because they also highlight the cases in Table 1 that have Nd=0 or 1, which are themselves especially significant cases, (13) and (15). One can see at a glance in Table 1 what ductility state exists for a given materials type under the specified stress state.

	$\frac{T}{C}=0$	$\frac{T}{C}=\frac{1}{3}$	$\frac{T}{C}=\frac{1}{2}$	$\frac{T}{C}=\frac{2}{3}$	$\frac{T}{C}=1$
Uniaxial Compression	0	1	3/2	2	3
Shear	-1	0	1/2	1	2
Uniaxial Tension	-1	-1/3	0	1/3	1

Table 1 Ductility numbers, Nd

Several materials types cases will now be given for the three basic stress states. The classifications into the four failure types will be given in accordance with (12)-(15) and Fig. 1.

Dolomite, T/C=1/15

Simple Compression
Nd=1/5   Ductile

Simple Shear
Nd=-4/5   Brittle

Simple Tension
Nd=-13/15   Brittle

Ceramic, T/C=1/5

Simple Compression
Nd=3/5   Ductile

Simple Shear
Nd=-2/5   Brittle

Simple Tension
Nd=-3/5   Brittle

Cast Iron, T/C=1/3

Simple Compression
Nd=1   Full Ductility

Simple Shear
Nd=0   D/B Transition

Simple Tension
Nd=-1/3   Brittle

Glassy Polymer, T/C=1/2

Simple Compression
Nd=3/2   Full Ductility

Simple Shear
Nd=1/2   Ductile

Simple Tension
Nd=0 D/B Transition

Epoxy, T/C=2/3

Simple Compression
Nd=2 Full Ductility

Simple Shear
Nd=1 Full Ductility

Simple Tension
Nd=1/3 Ductile

Perfectly Ductile Metal, T/C=1

Simple Compression
Nd=3 Full Ductility

Simple Shear
Nd=2 Full Ductility

Simple Tension
Nd=1 Full Ductility

Two other more complex but still basic stress states will now be considered, that of eqi-biaxial stresses and that of eqi-triaxial tension.

For eqi-biaxial stresses there is

$$Nd = 1 + \frac{T}{C} \pm 2\sqrt{1 - \frac{T}{C} + \left(\frac{T}{C}\right)^2} \qquad (16)$$

The plus sign is for compression and the minus sign is for tension. Eqi-biaxial compression is always full ductility for all materials. Eqi-biaxial tension is at the ductile/brittle transition at T/C=1 and otherwise it is brittle.

For eqi-triaxial tension

$$Nd = 3\frac{T}{C} - \frac{1}{1-\frac{T}{C}} \tag{17}$$

This is always brittle for all materials types.

Finally, it should be mentioned that there are limitations to this ductility measure, Nd. Although it applies to all the common and major materials classes, there are specialty materials for which it doesn't appear to be applicable. The most obvious example is that of amorphous metals. As shown by Christensen, Li, and Gao [10] those materials don't show so much of a ductile/brittle transition behavior as they do that of a failure modes transition. The failure modes transition is completely general whereas that of the ductile/brittle transition specialization of it can be somewhat more exclusionary in application.

5. Ductile Versus Brittle Designations Across the Full Range of Major Materials Types and All Stress States

The ductility number Nd (6) and its conditions (12)-(15) will be used to build up the ductile versus brittle failure designations for a very wide variety of stress states and materials types specified by their T/C ratios. This spread of failure behaviors is shown in Table 2.

	$\frac{T}{C}=0$	$\frac{T}{C}=\frac{1}{3}$	$\frac{T}{C}=\frac{1}{2}$	$\frac{T}{C}=\frac{2}{3}$	$\frac{T}{C}=1$
Eqi-Triaxial Compression	No Failure	No Failure	No Failure	No Failure	No Failure
Eqi-Biaxial Compression	Full Ductility	Full Ductility	Full Ductility	Full Ductility	Full Ductility
Uniaxial Compression	D/B Transition	Full Ductility	Full Ductility	Full Ductility	Full Ductility
Simple Shear	No Strength	D/B Transition	Ductile	Full Ductility	Full Ductility
Uniaxial Tension	No Strength	Brittle	D/B Transition	Ductile	Full Ductility
2:1 Biaxial Tension	No Strength	Brittle	Brittle	D/B Transition	Ductile
Eqi-Biaxial Tension	No Strength	Brittle	Brittle	Brittle	D/B Transition
Eqi-Triaxial Tension	No Strength	Brittle	Brittle	Brittle	Brittle

Table 2   Ductile and full ductility versus brittle stress states and materials types

The ductile and full ductility and brittle designations are used everywhere in Table 2.   This array of results shows the consistency of the failure types to primarily group into ductile versus brittle character based upon the stress state type and the materials type designated by T/C values. Generally compressive stress states are predominately ductile whereas generally tensile stress states are predominately brittle. Similarly, generally low T/C materials are predominately brittle whereas generally high T/C materials are predominately ductile. Table 2 provides certainty on all possible combinations of the four failure types.

It is interesting that in the ductile and full ductility side of the ledger in Table 2 only three cases are at the “ductile” level while the other twelve cases come in at the “full ductility” level. It is the ductile/brittle transition that enforces order and organization into this complete array of the dependence on the stress states and the materials types.

It is further revealing to tighten the range of cases to be considered. From Table 2 take the cases from uniaxial compression to eqi-biaxial tension. These are the cases that explicitly include the ductile/brittle transition within its extremities from T/C=0 to T/C=1. Secondly take the behavior of “no strength” as being equivalent to “brittle” behavior as would occur at very small values of T/C approaching zero. With these two physical conditions or restrictions, the ductile (including full ductility) and brittle failure behaviors in Table 2 take the forms shown in Table 3.

	$\frac{T}{C}=0$	$\frac{T}{C}=\frac{1}{3}$	$\frac{T}{C}=\frac{1}{2}$	$\frac{T}{C}=\frac{2}{3}$	$\frac{T}{C}=1$
Uniaxial Compression	D/B Transition	Ductile	Ductile	Ductile	Ductile
Simple Shear	Brittle	D/B Transition	Ductile	Ductile	Ductile
Uniaxial Tension	Brittle	Brittle	D/B Transition	Ductile	Ductile
2:1 Biaxial Tension	Brittle	Brittle	Brittle	D/B Transition	Ductile
Eqi-Biaxial Tension	Brittle	Brittle	Brittle	Brittle	D/B Transition

Table 3 Ductile/brittle perfect anti-symmetry

The results in Table 3 show diagonal anti-symmetry surrounding the ductile/brittle transitions. There is perfect order and organization in Table 3. The case of uniaxial tension at T/C=1/2 is the central focal point of all that transpires in Table 3. Any particular stress state is just as important and dominant as the materials type, T/C. They are equally determinative in forming a rational answer to the crucial question: is the failure type expected to be of the rather benign ductile progression of failure or of the sudden and destructive brittle failure? Table 2 and Table 3 have some of the same appeal as in the Periodic Table in assuring a considerable degree of understanding for these extraordinarily complex physical processes.

6. Competition Between a Ductile Failure Mechanism and a Brittle Failure Mechanism

An adhesive joint supporting both normal stress and shear stress states will be used to illustrate ductile/brittle failure response characteristics. The configuration of the joint is shown in Fig. 2.

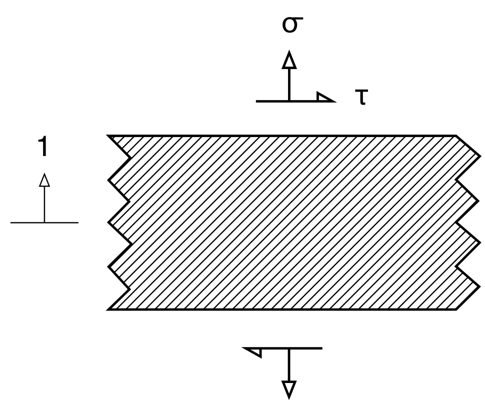

Fig. 2 Adhesive joint

For the applied normal stress term, the associated strains are given by

$$\begin{aligned} &\varepsilon_{11} \neq 0 \\ &\varepsilon_{22} = \varepsilon_{33} = 0 \end{aligned} \tag{18}$$

The corresponding stresses are

$$\begin{aligned} &\sigma_{11} = \sigma \\ &\sigma_{22} = \sigma_{33} = \frac{\nu}{1-\nu}\sigma \\ &\sigma_{ii} = \left(\frac{1+\nu}{1-\nu}\right)\sigma \end{aligned} \tag{19}$$

The stresses (19) are to be combined with the shear stress τ shown in Fig. 2.

The failure criterion (4) then takes the form

$$\left(\frac{1+\nu}{1-\nu}\right)\left(1-\frac{T}{C}\right)\hat{\sigma}+\left(\frac{1-2\nu}{1-\nu}\right)\hat{\sigma}^2+3\hat{\tau}^2=\frac{T}{C} \tag{20}$$

Typical polymeric adhesive properties will be used in (20). An aerospace grade epoxy has about the values

$$\begin{aligned}\frac{T}{C}&=\frac{2}{3}\\ \nu&=\frac{1}{3}\end{aligned} \tag{21}$$

Combining (21) into (20) gives the solution as

$$\hat{\tau}=\pm\frac{1}{6}\sqrt{8-8\hat{\sigma}-3\hat{\sigma}^2} \tag{22}$$

The ductility number values follow from (6), (19) and (21) as

$$Nd=1-2\hat{\sigma} \tag{23}$$

These failure stresses and ductile/brittle behaviors are as shown in Fig. 3 for positive values of $\hat{\sigma}$. The failure curve extends much further to the left and it is fully ductile in that region.

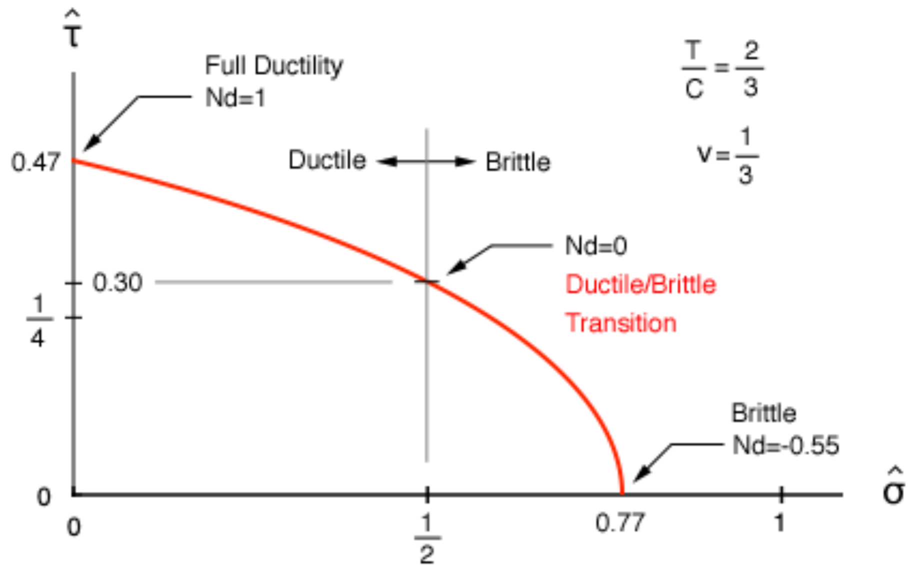

Fig. 3 Failure stresses and ductility levels in the adhesive joint

Different combinations of the transverse normal stress and the shear stress terms in Fig. 3 lead to very large differences in the ductile/brittle character of the failure event in this important and typical practical application.

7. The Takeaway

There are two main takeaways coming from the present work:

1. The quantitative characterization of the ductility of materials failure has been developed and applied in detail.

2. The full documentation has been given for the many verifications of the failure theory.

The second area speaks for itself. The first takeaway will be elaborated upon and finalized here since it is so original in concept, new in presentation, and unique in derivation.

The ductlity number, Nd, differentiates between brittle failure in one broad condition and a gradation scale of ductility's in the opposite, broad class. Beyond the range of the graded scales of ductility lies the state of full, perfect ductility. Wherever a particular case lies depends upon the stress state and the materials type as specified by the materials T/C strengths ratio. Nothing more is needed. This provides the rigorous characterization of the state of failure for solids as contrasted with the vague and nebulous state of turbulence for fluids.

One need only look at the results for the uniaxial tensile testing to failure to see the progression in the degrees of ductility. That much is well known and has been accepted heuristically as long as materials have been developed for performance purposes. In the present context the state of brittleness is just as interesting and important as are the degrees of ductility. Maybe it is even more important from the point of view of the hazards that brittleness imposes.

The ductility number concept presents brittleness as an active or non-active behavior. That is, the well posed materials failure problem leads to the condition of being either brittle or in an alternative as being ductile by degrees of intensity and effectiveness. Is there data to support the condition of brittleness as being a total or nothing behavior? The term "nothing" in the brittleness context means that even though there are degrees of ductility there are not degrees of brittleness. Brittleness is the total absence of ductility. It will be helpful to consider some specific cases.

Glassy polymers have T/C as about 1/2. Glassy polymers do show brittle behavior with the fracture plane at 90 degrees to the axis in uniaxial tension, Ref. [5]. For uniaxial tension the ductile/brittle transition is exactly at T/C=1/2. Here the all brittle behavior begins at or near Nd=0.

What about other materials with their T/C values near to 0 or near to 1. A very ductile metal with T/C=1 shows the state of eqi-biaxial tension as being at the ductile/brittle transition. Brittle behavior is shown by the fragmentation behavior of spherical pressure vessels with no evidence of extensive strain hardening. Thus the complete brittle behavior emerges here for a perfectly ductile metal at the ductile/brittle transition in eqi-biaxial tension.

For the third case with T/C near to 0 the ductile/brittle failure theory predicts that the ductile/brittle transition is at T/C=0 in uniaxial compression. In agreement with this predicted behavior geological materials with very small values of T/C show a brittle splitting failure mode in uniaxial compression. Thus the failure is of the all brittle type beginning near Nd=0.

On close inspection these three examples reveal the successful prediction of the conditions at which brittle failure commences to become a very significant problem. Further, these examples represent and cover the full range of the different major materials classes, from the very ductile extreme to the very brittle extreme. Such matters have always posed open-ended questions that seemed almost impossibly difficult to answer. With this work that is no longer true, if it ever was.

The brittle aspects and the ductile aspects of observed failure behaviors show consistency with the theoretical predictions. It can now be stated that the failure theory development is completed and it includes a precise methodology for treating the complications and full implications of the ductile versus the brittle failure mechanisms.

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